Turbulence in Pipes: The Moody Diagram and Moore's Law

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Thank you

- Mark Zagarola, Beverley McKeon, Rongrong Zhao, Michael Shockling, Richard Pepe, Leif Langelandsvik, Marcus Hultmark, Juan Jimenez
- James Allen, Gary Kunkel, Sean Bailey
- Tony Perry, Peter Joubert, Peter Bradshaw, Steve Orszag, Jonathan Morrison, Mike Schultz

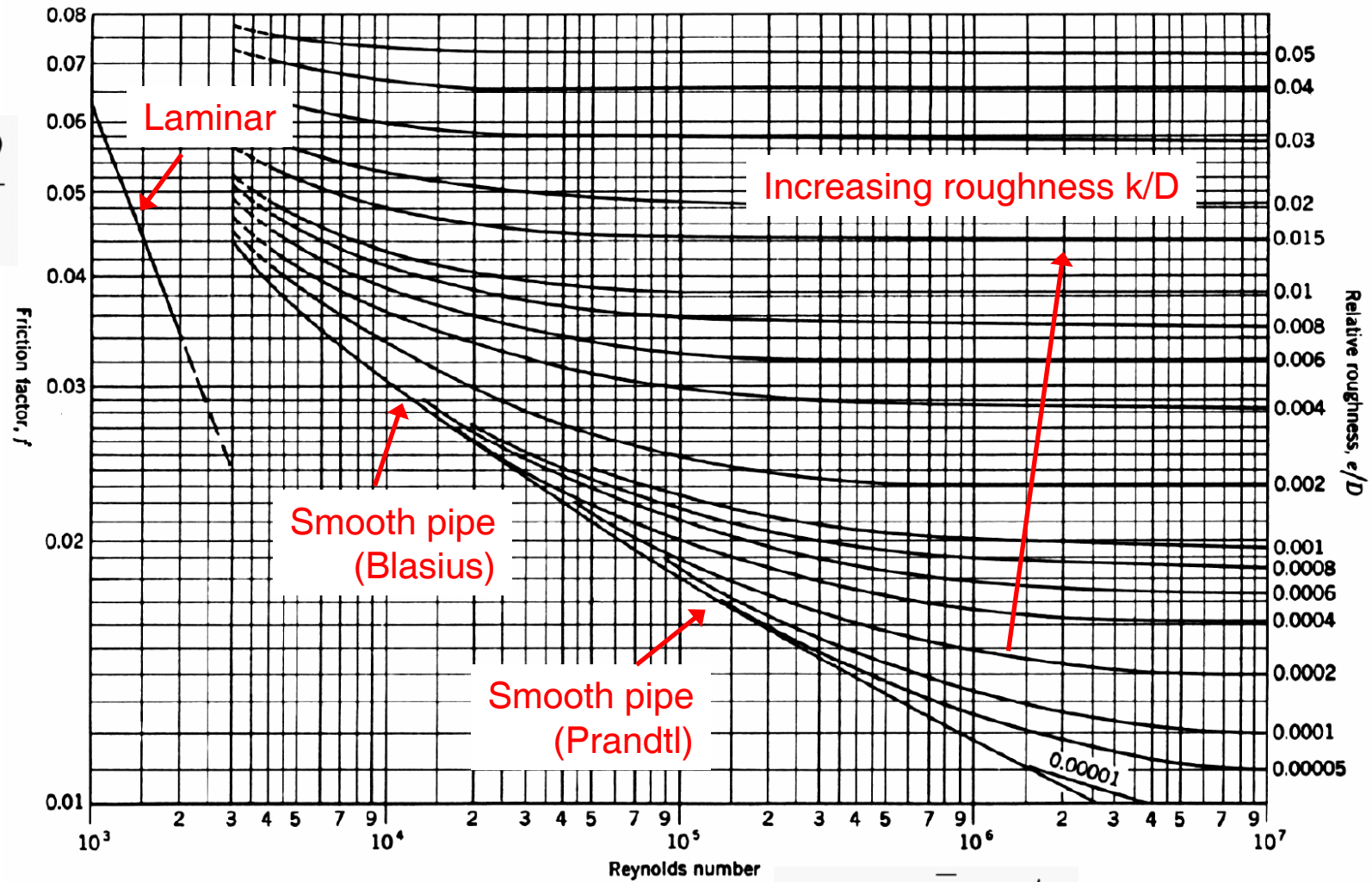
Lewis Ferry Moody 1880-1953

- Professor of Hydraulic Engineering at Princeton University, 1930-1948
- "Friction factors for pipe flow," Trans. of the ASME, 66, 671-684, 1944

(from Glenn Brown, OSU)

The Moody Diagram

$$\lambda \equiv \frac{-\frac{dp}{dx} D}{\frac{1}{2}\rho \overline{U}^2}$$



$$\mathrm{Re} = \bar{U} D / \nu$$

Outline

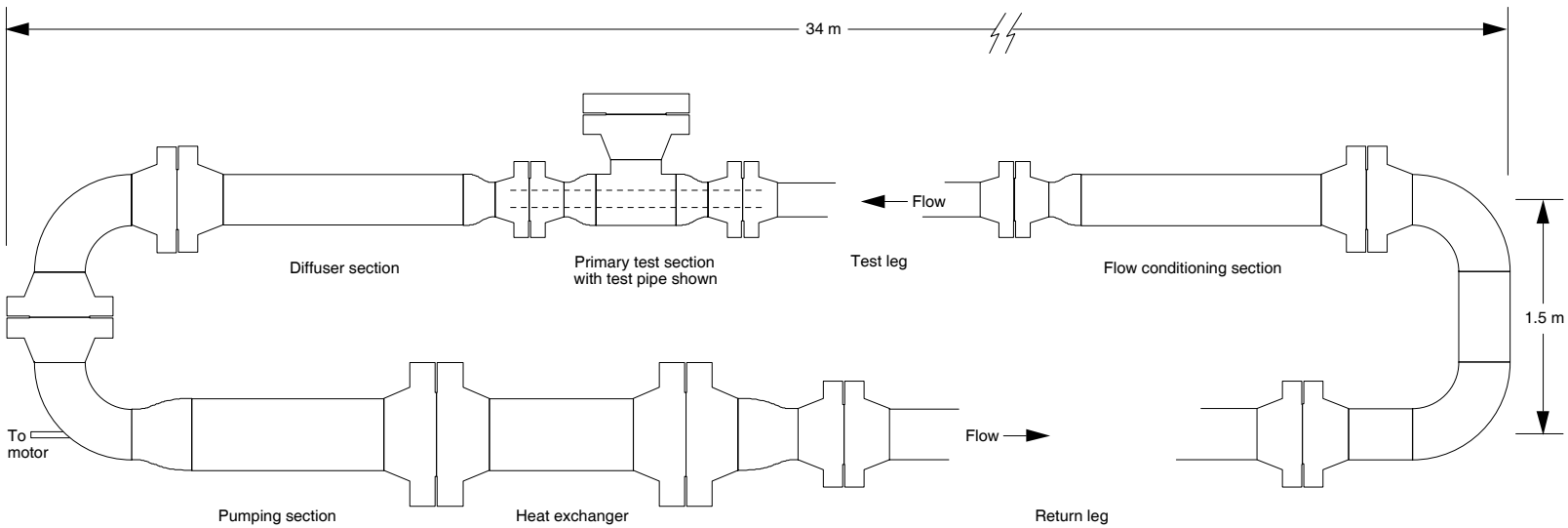
- Smooth pipe experiments
  - $Re_D = 31 \times 10^3$ to 35×10^6
- Rough pipe experiments
  - Smooth to fully rough in one pipe
  - Honed surface roughness
  - Commercial steel pipe roughness
- A new Moody diagram(s)?
- Predicting arbitrary roughness behavior
  - Theory
  - Petascale computing

High Reynolds number facilities

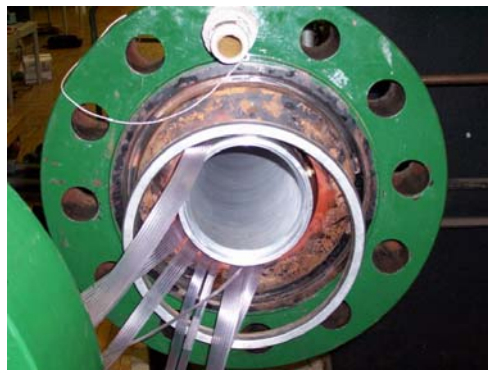
80’ x 120’
NASA-Ames

National Transonic Facility, NASA-LaRC

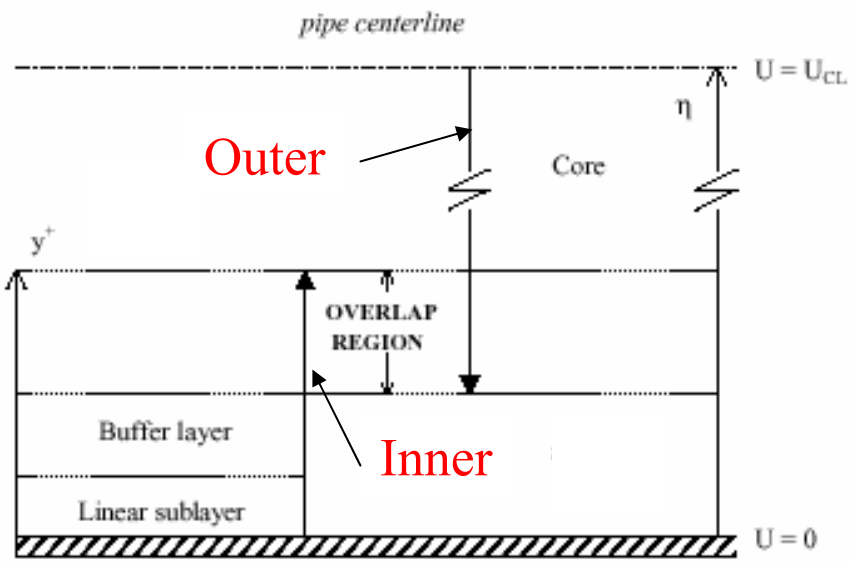
High Reynolds Number in the lab:
Compressed air up to 200 atm as the working fluid



Princeton/DARPA/ONR Superpipe:
Fully-developed pipe flow
$Re_D = 31 \times 10^3$ to 35×10^6

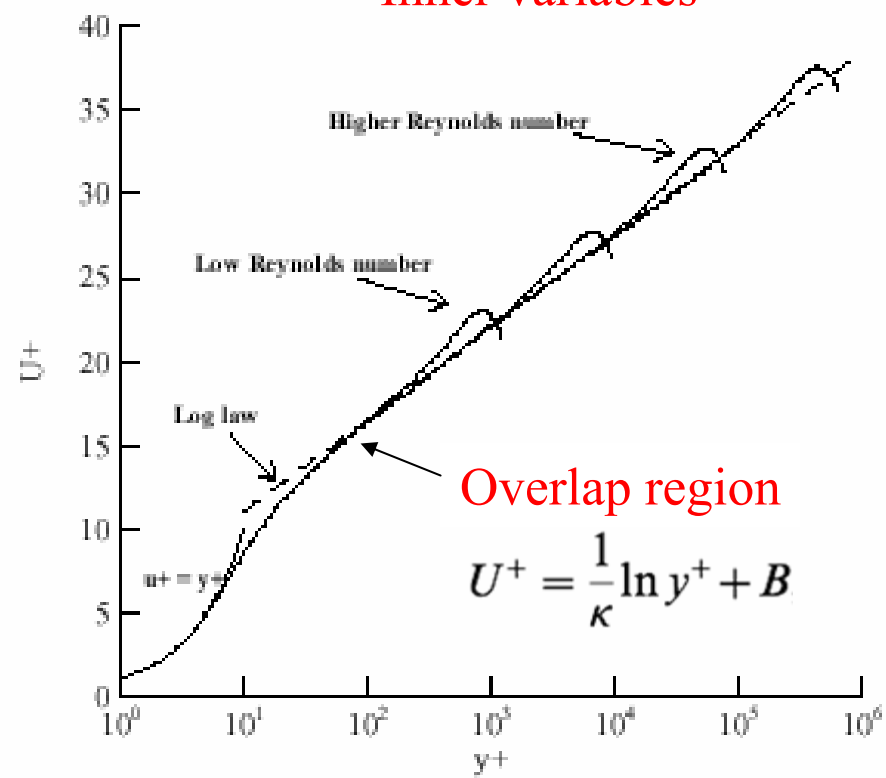


Standard velocity profile



$U^+ = U/u_\tau$

Inner variables



$y^+ = yu_\tau/\nu$

$$\lambda = \frac{4\tau_w}{\frac{1}{2}\rho\overline{U}^2} = 8\left(\frac{u_\tau}{\overline{U}}\right)^2$$

$$u_\tau = \sqrt{\tau_w/\rho},$$

Similarity analysis for pipe flow

Incomplete similarity (in Re) for inner & outer region

$$\frac{U}{u_\tau} = U^+ = f\left(\frac{yu_\tau}{\nu}, \frac{Ru_\tau}{\nu}\right) = f\left(y^+, R^+\right)$$ Inner scaling

$$\frac{U_{CL} - U}{u_o} = g\left(\frac{y}{R}, \frac{Ru_\tau}{\nu}\right) = g\left(\eta, R^+\right)$$ Outer scaling

Complete similarity (in Re) for inner & outer region

$$U^+ = f\left(\frac{yu_\tau}{\nu}\right) = f\left(y^+\right)$$ Inner

$$\frac{U_{CL} - U}{u_o} = g\left(\frac{y}{R}\right) = g(\eta)$$ Outer

Overlap analysis: two velocity scales

At low Re:

$$\frac{u_o}{u_\tau} = h\left(R^+\right)$$

> Match velocities and velocity gradients $\Rightarrow$ power law

At high Re:

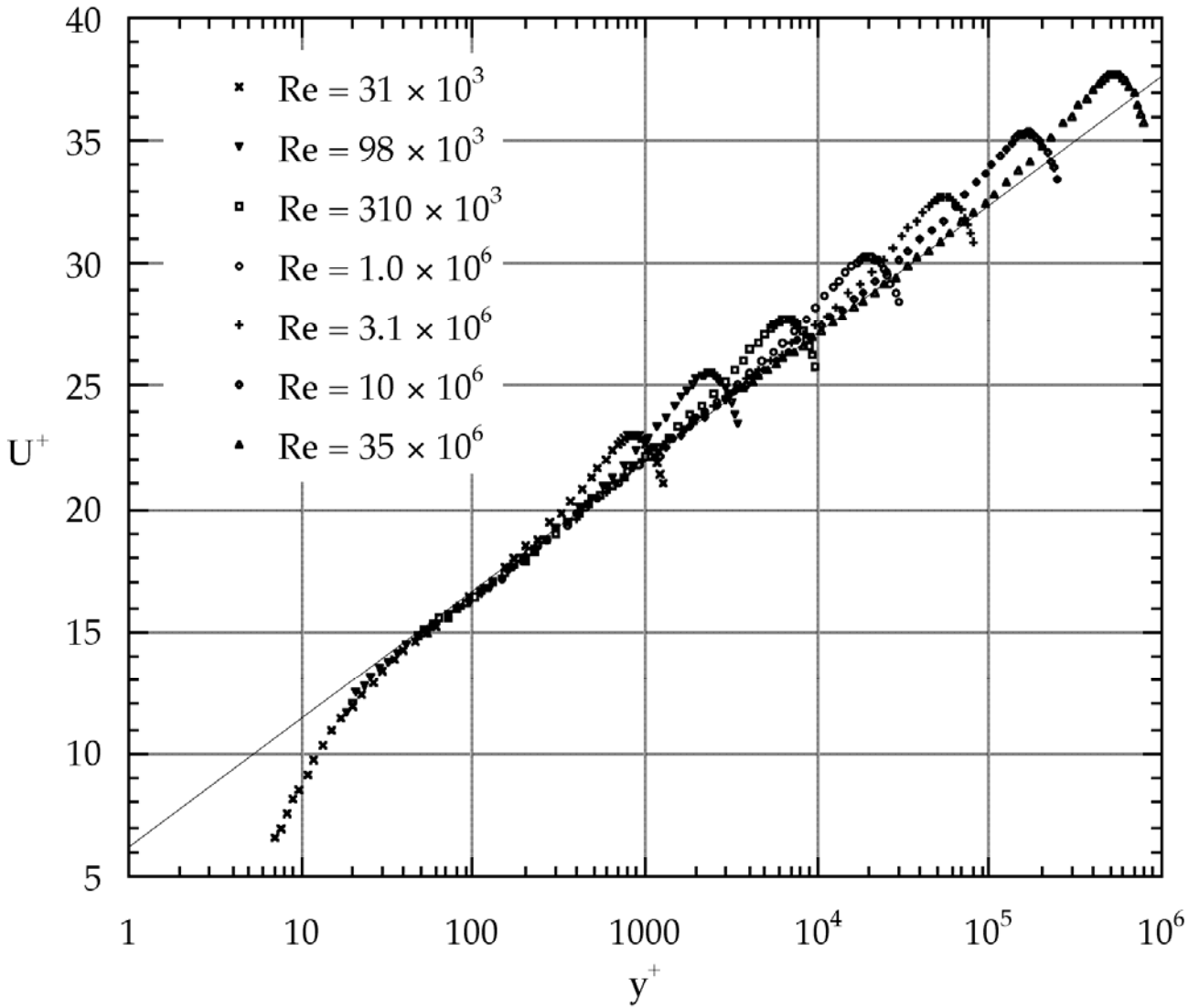
$$\frac{u_o}{u_\tau} = \text{constant}$$

> Match velocities and velocity gradients $\Rightarrow$ power law

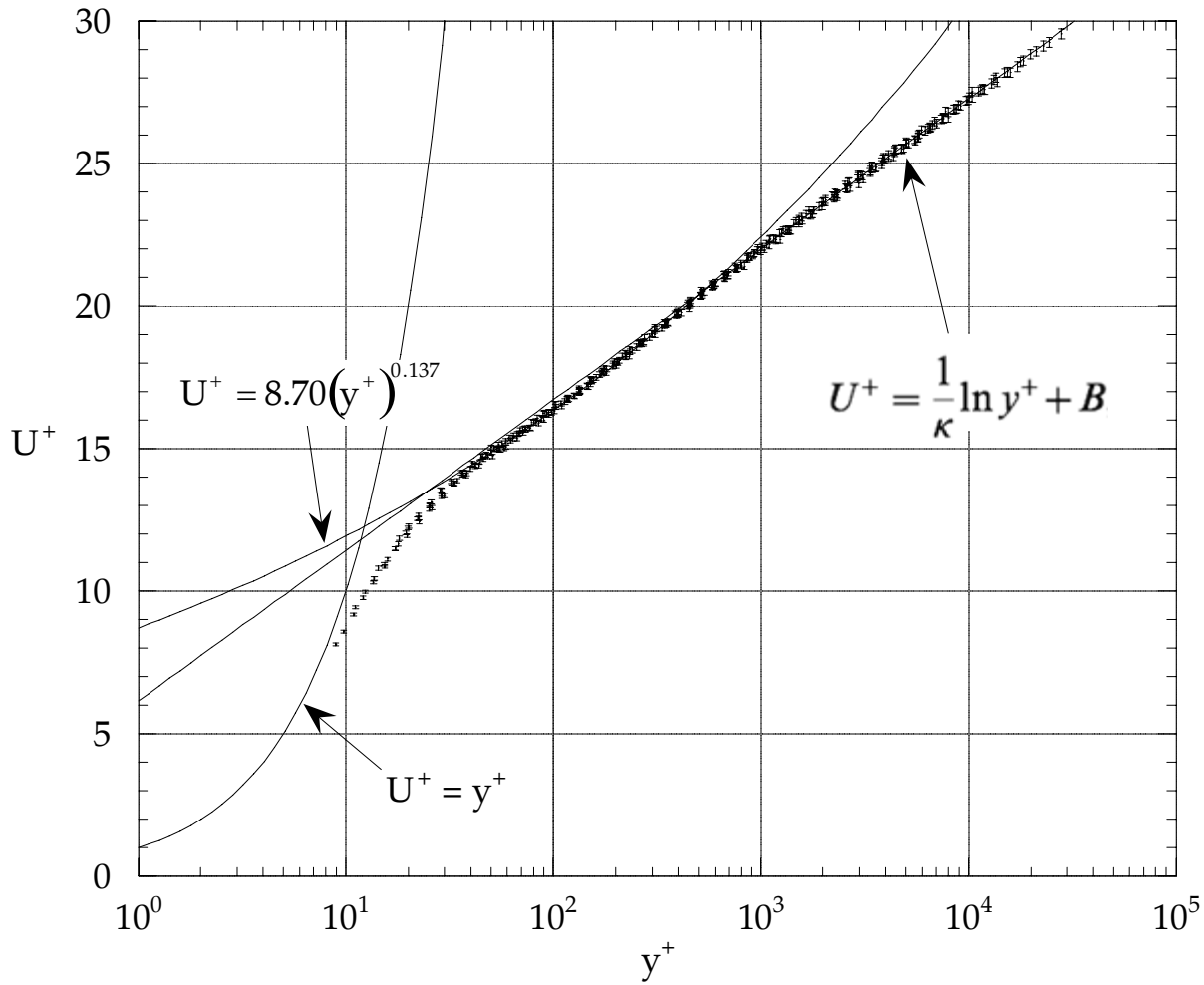
> Match velocity gradients $\Rightarrow$ log law

$$U^+ = \frac{1}{\kappa}\ln y^+ + B$$

Superpipe results



Pipe flow inner scaling



Smooth pipe summary

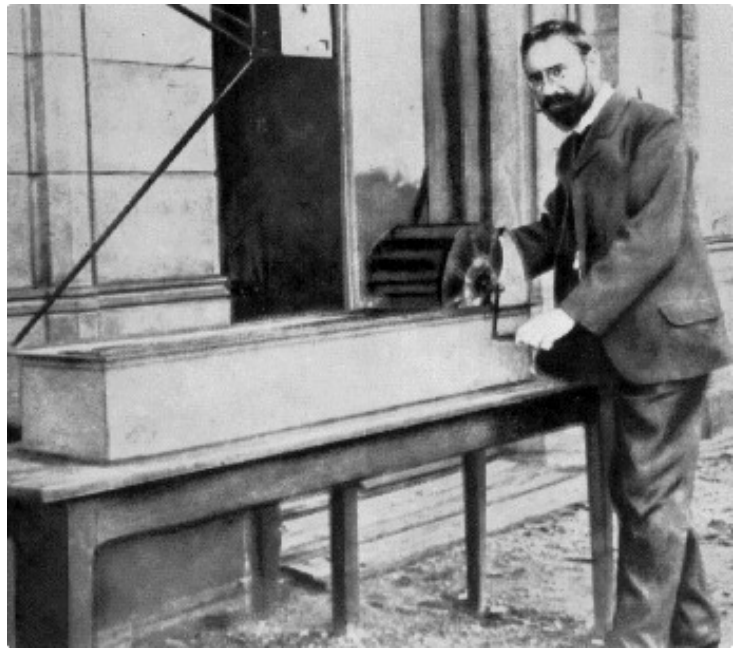
- Log law only appears at sufficiently high Reynolds number
- New log law constants: κ=0.421, B=5.60 (cf. 0.41 and 5.0)
- Spalart: $\Delta\kappa$ = 0.01, gives ΔC_D=1% at flight Reynolds numbers
- New outer layer scaling velocity for "low" Reynolds number

What about the friction factor? Need to integrate velocity profile.

$$\lambda = \frac{4\tau_w}{\frac{1}{2}\rho\overline{U}^2} = 8\left(\frac{u_\tau}{\overline{U}}\right)^2 \qquad U^+ = \frac{1}{\kappa}\ln y^+ + B$$

Gottingen, Germany

Theodor von Kármán



Ludwig Prandtl

Johann Nikuradse, 1933
(from Glenn Brown)

Prandtl's "Universal Friction Law"

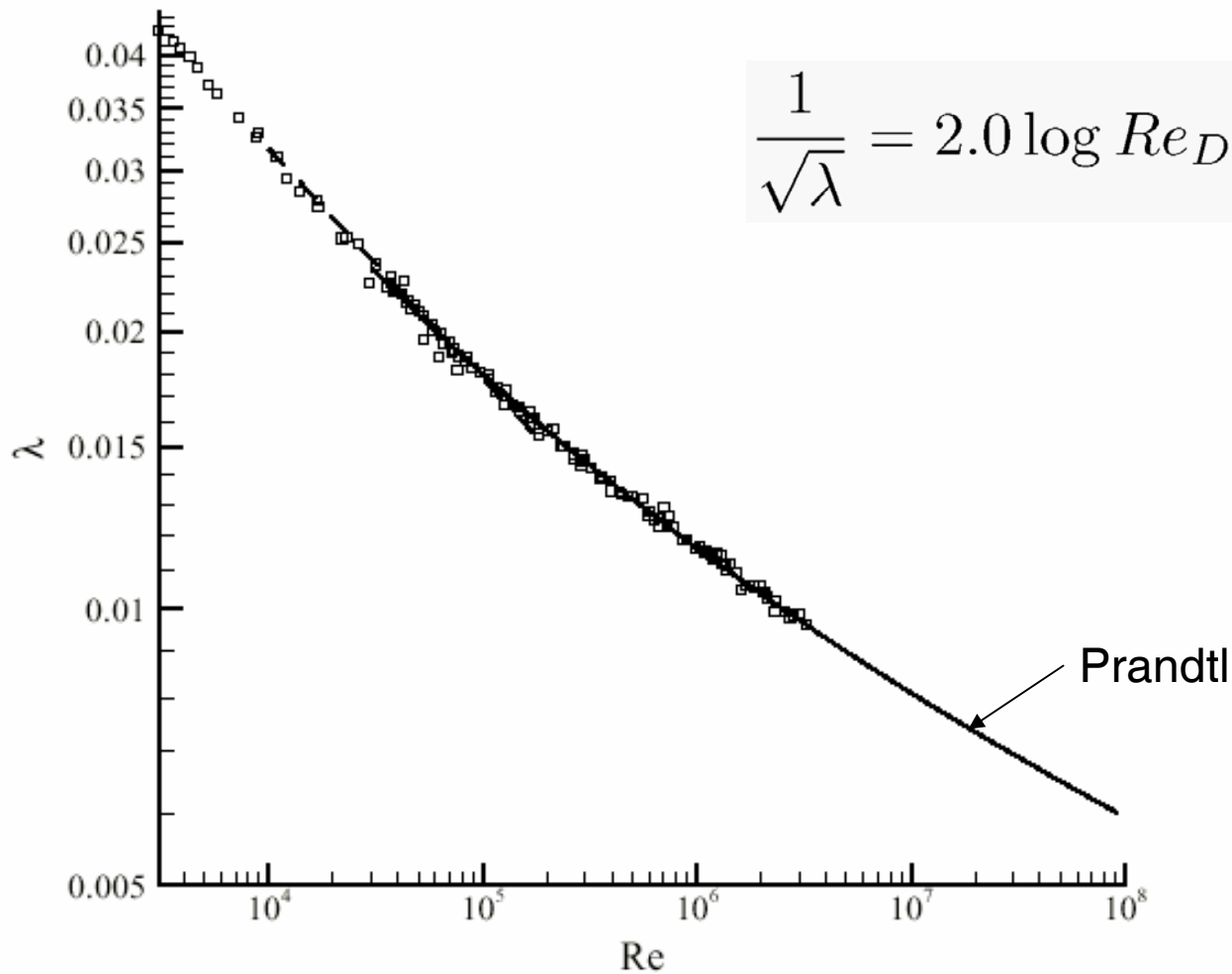
With $\kappa = 0.4$, and $B = 5.5$ (Nikuradse), and $C_3 = C_4 = 0$:

$$\frac{1}{\sqrt{\lambda}} = 2.035 \ln Re\sqrt{\lambda} - 0.913$$

Prandtl:

$$\frac{1}{\sqrt{\lambda}} = 2.0 \log Re_D\sqrt{\lambda} - 0.8$$

Nikuradse's data



$$\frac{1}{\sqrt{\lambda}} = 2.0 \log Re_D \sqrt{\lambda} - 0.8$$

Figure 1.2: Comparison of the Blasius relationship (−−), Prandtl's friction factor relationship (− · −) and the friction factor data of Nikuradse (which was used to formulate the empirical expression).

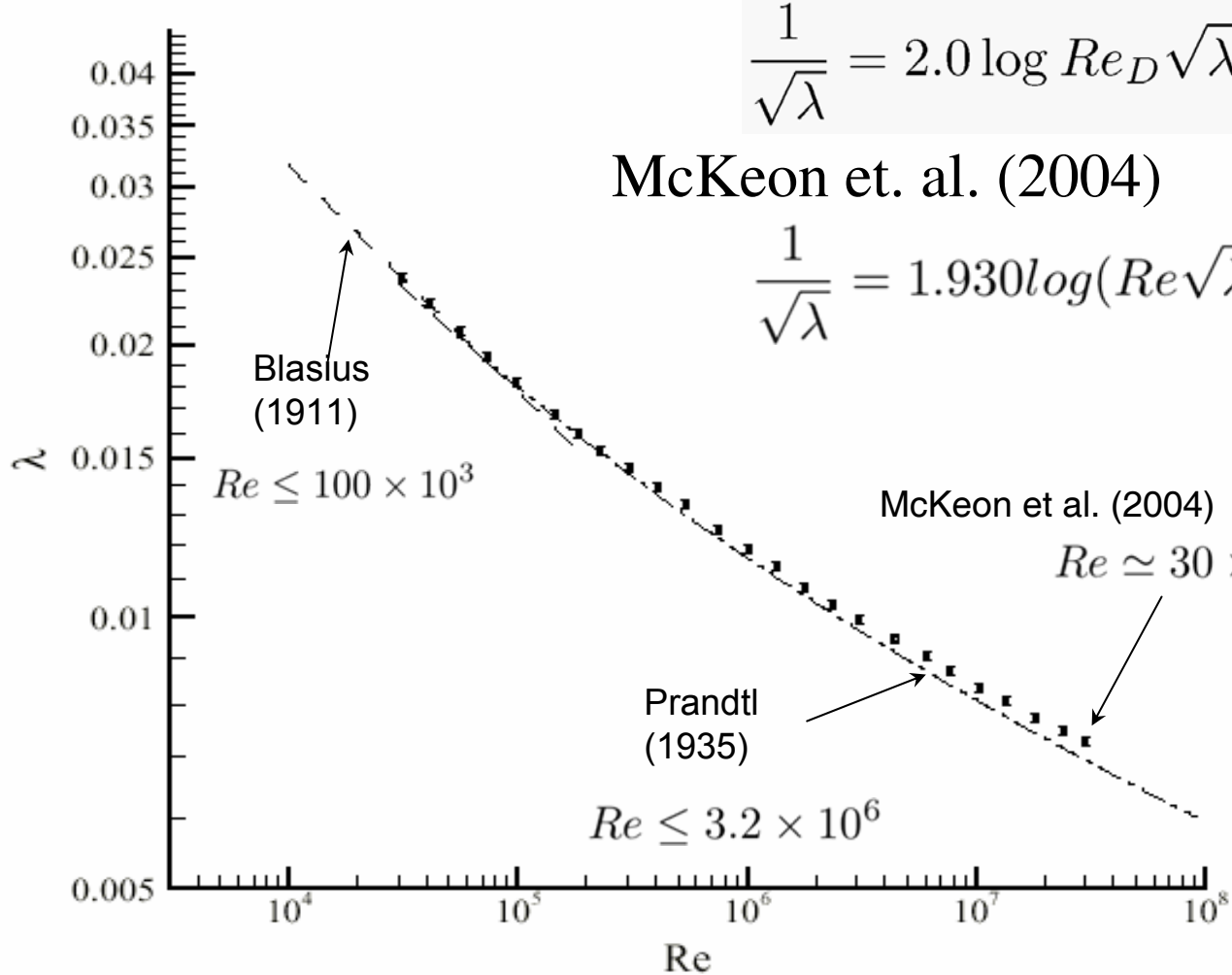
Superpipe results

Prandtl (1935)

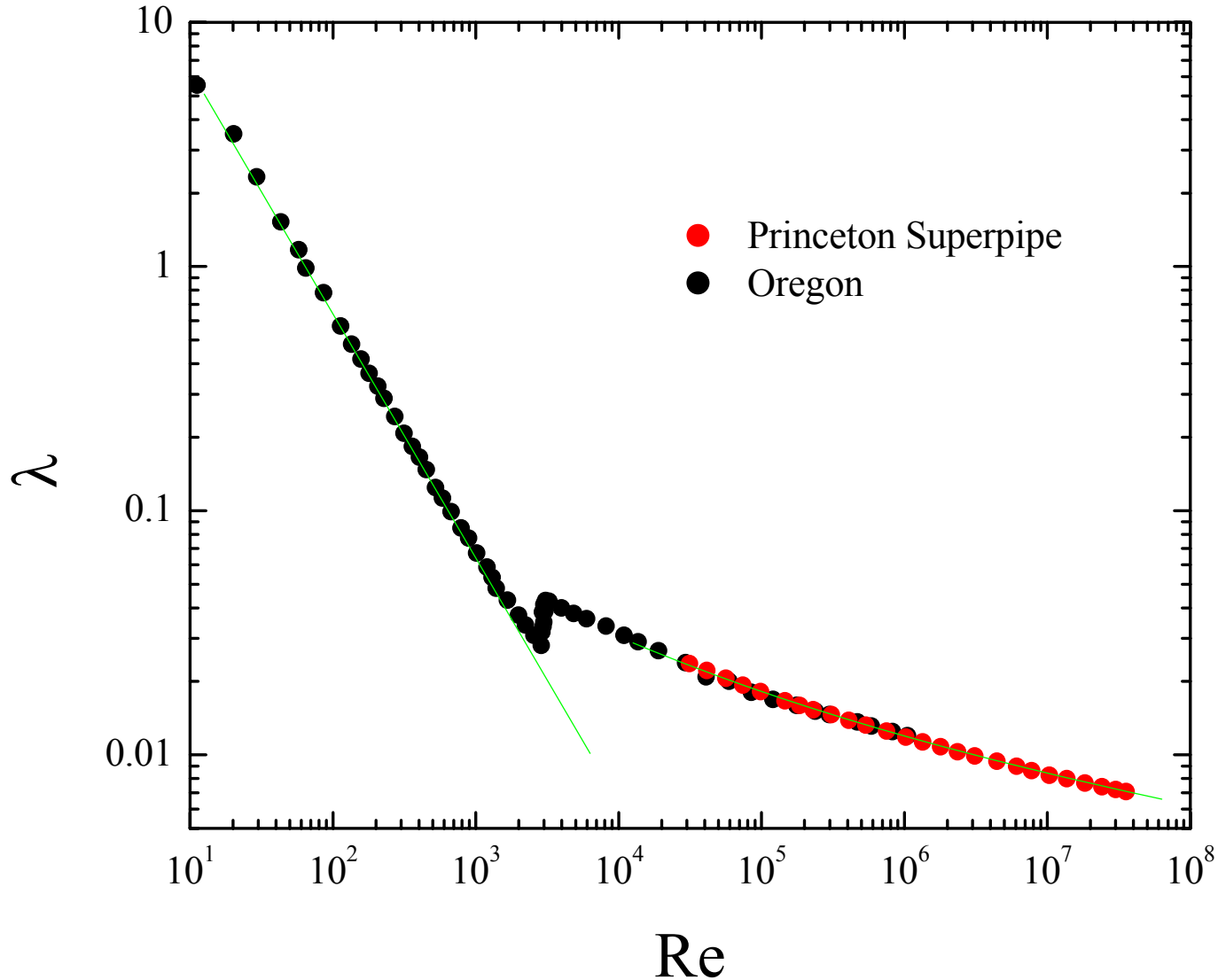
$$\frac{1}{\sqrt{\lambda}} = 2.0 \log Re_D \sqrt{\lambda} - 0.8$$

McKeon et. al. (2004)

$$\frac{1}{\sqrt{\lambda}} = 1.930 log(Re\sqrt{\lambda}) - 0.537$$



Two complementary experiments



Roughness

- How do we know the "smooth" pipe was really smooth at all Reynolds numbers?
- Were the higher friction factors at high Reynolds numbers evidence of roughness?

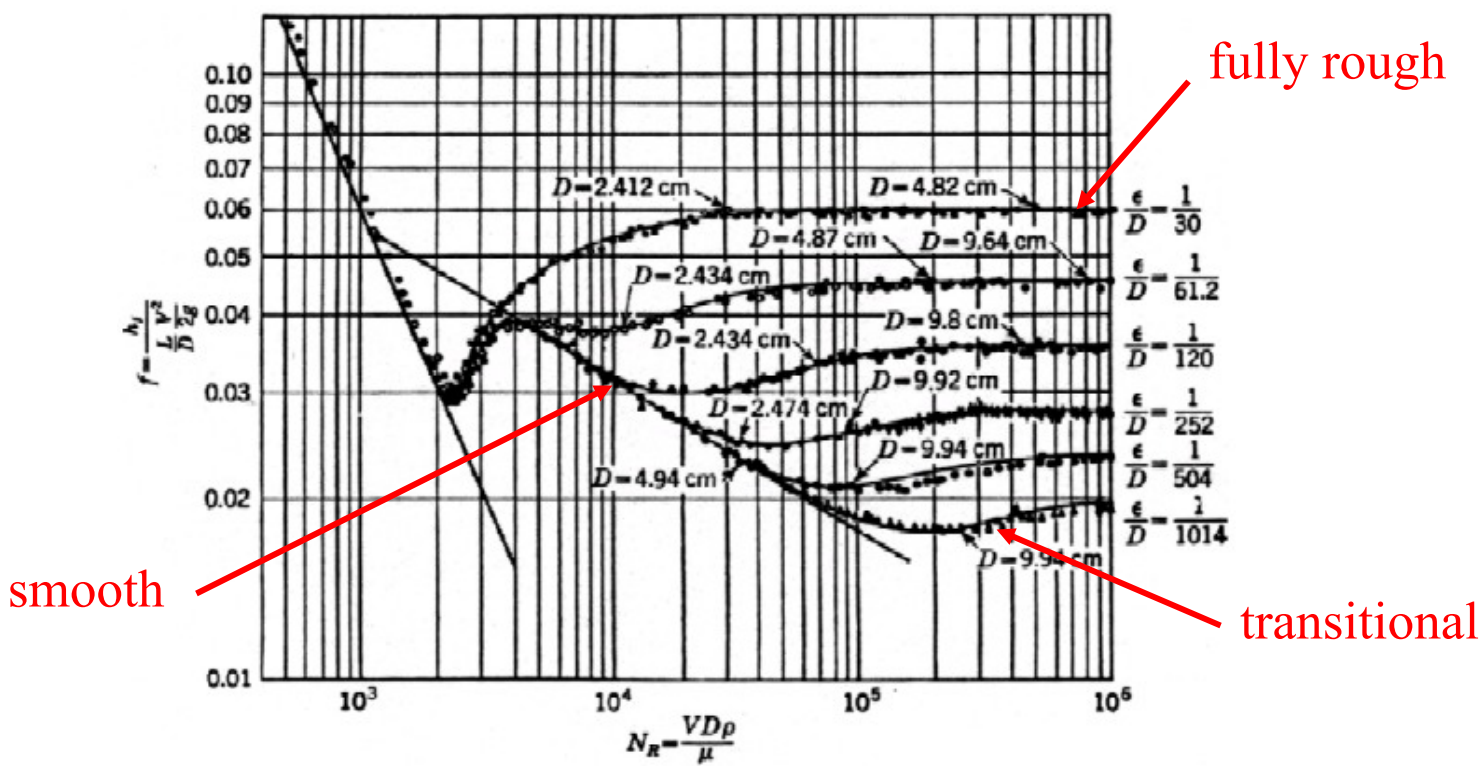
- What is k?
  - rms roughness height: k_{rms}
  - equivalent sandgrain roughness: k_s

- Nikuradse's rough pipe experiments (sandgrain roughness)
  - $k_s^+ < 5$, smooth
  - $5 < k_s^+ < 70$, transitionally rough
  - $k_s^+ > 70$, fully rough

$$k_s^+ = k_s u_\tau / \nu$$

Nikuradse's sandgrain experiments

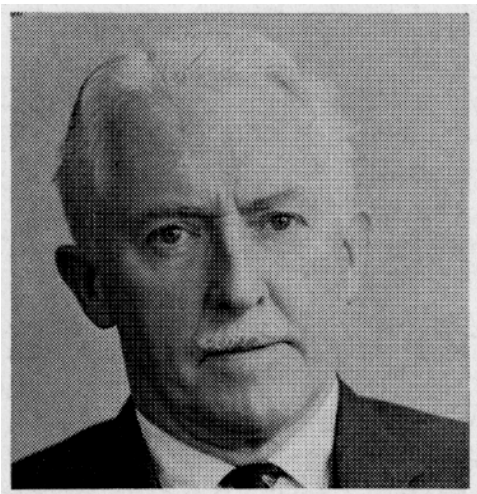
- Transition from smooth to fully rough included inflection



- "Quadratic Resistance" in fully rough regime - Reynolds number independence

Colebrook and the Moody Diagram

- Data from Colebrook & White (1938), Colebrook (1939)
- Tested various roughness types
  - Large and small elements
  - Sparse and dense distributions
- Studied many different pipes with “commercial” roughness
- Nikuradse sand-grain trend with inflection deemed irregular
- Focus on the behavior in the transitional roughness regime



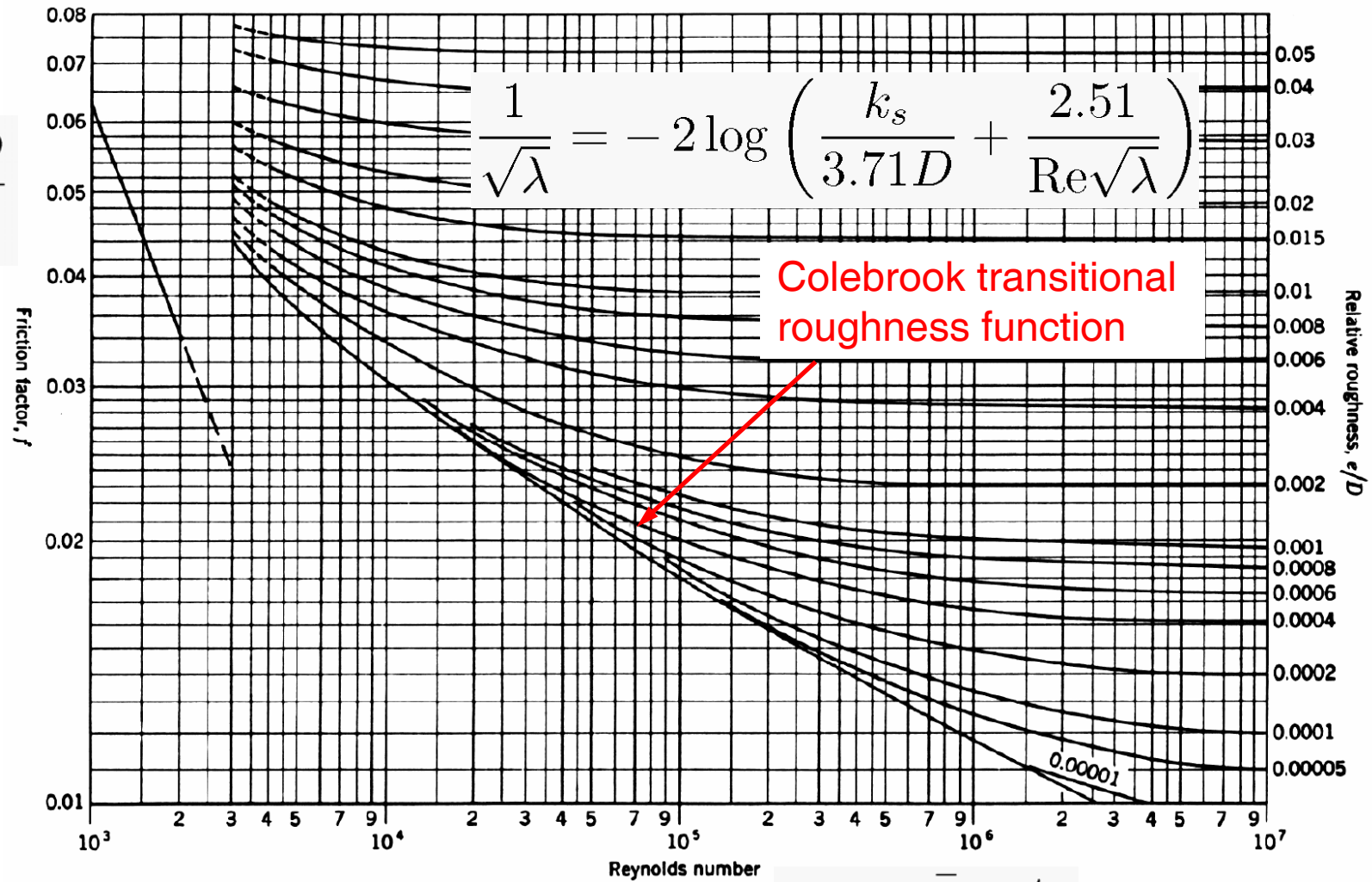
Cyril F. Colebrook

(from Glenn Brown)

Lewis Moody

The Moody Diagram

$$\lambda \equiv \frac{-\frac{dp}{dx}D}{\frac{1}{2}\rho\overline{U}^2}$$

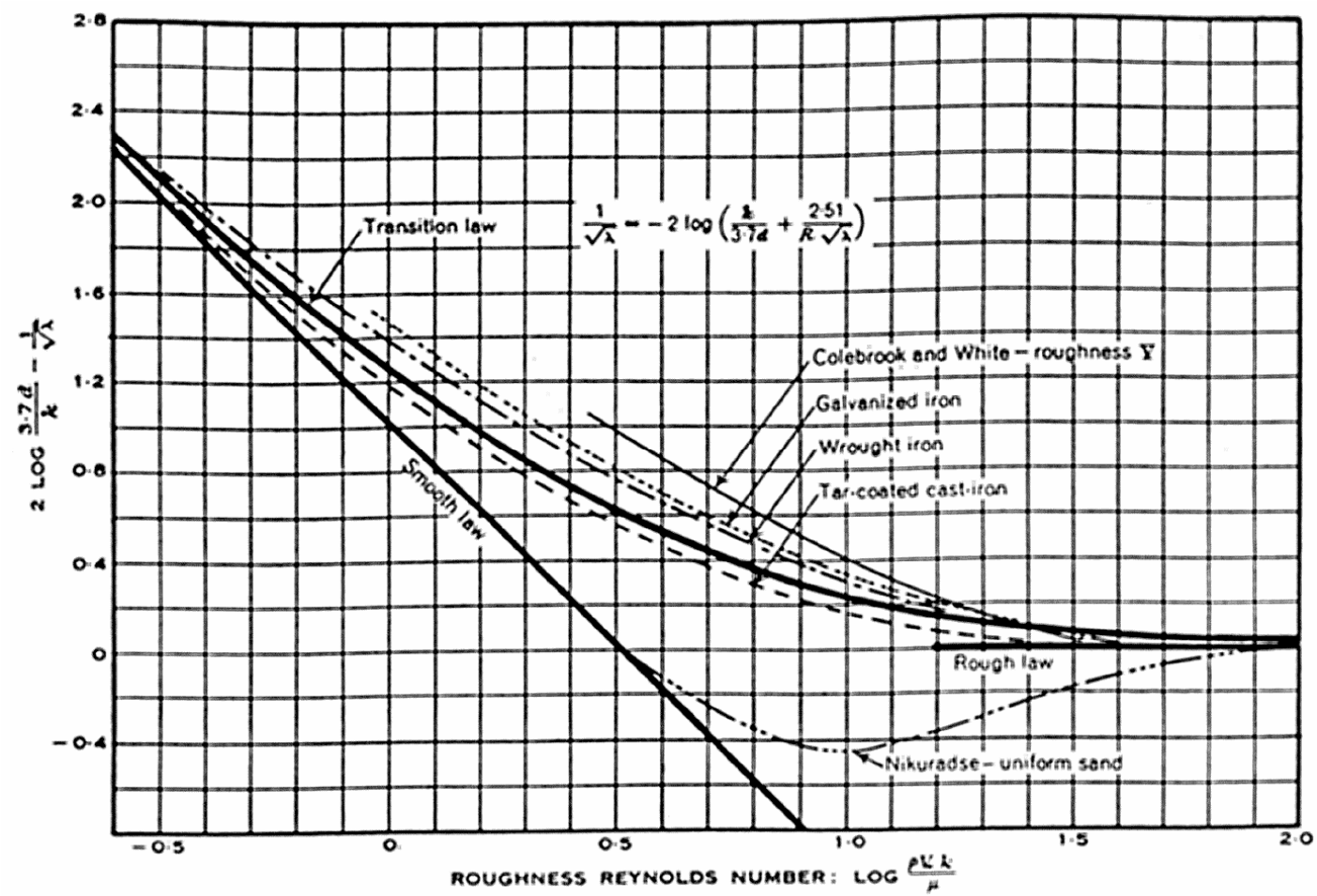


$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k_s}{3.71D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

Colebrook transitional roughness function

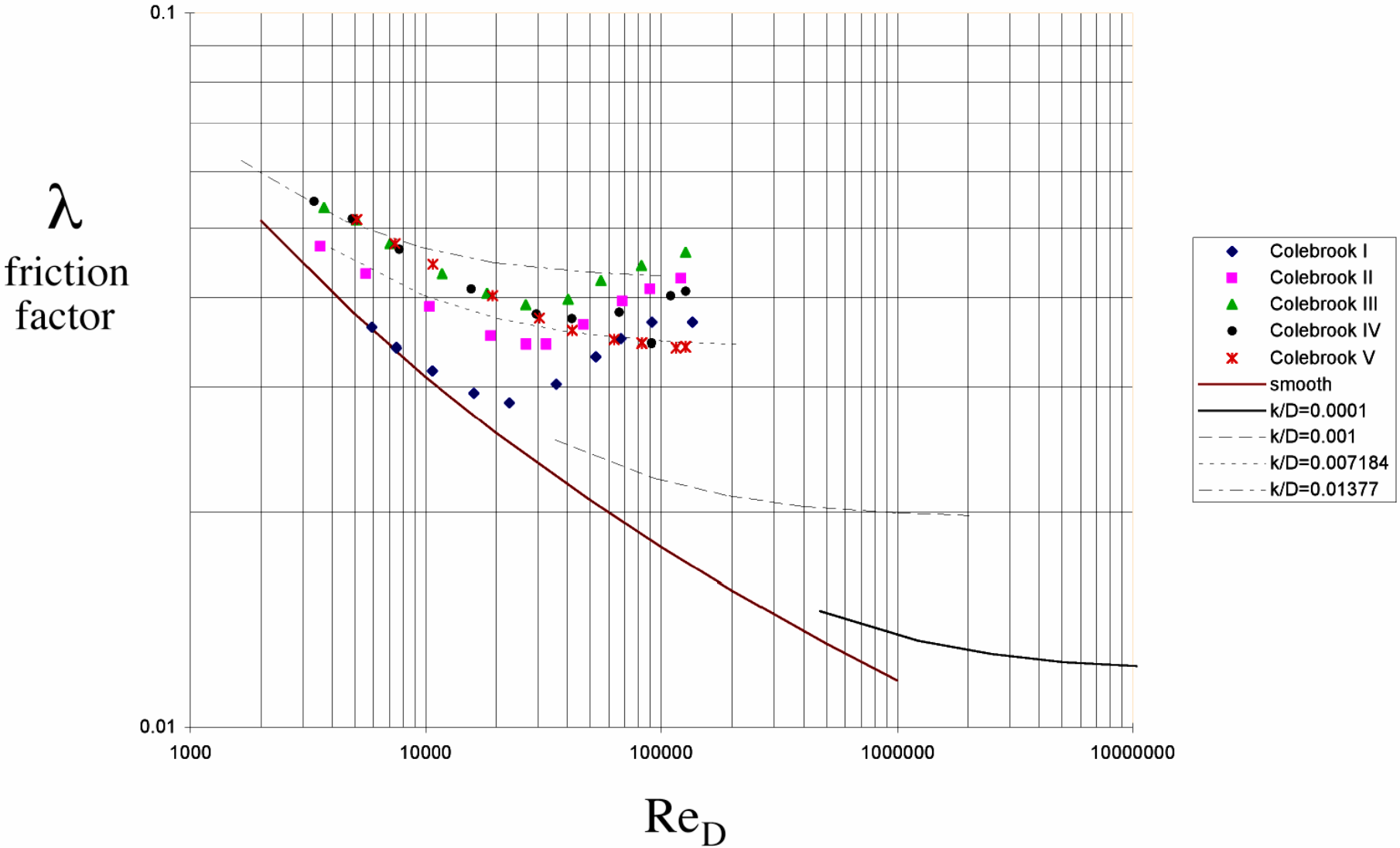
$$\mathrm{Re} = \bar{U}D/\nu$$

Where did Colebrook's function come from?



- Colebrook (1939)

Colebrook & White boundary layer results

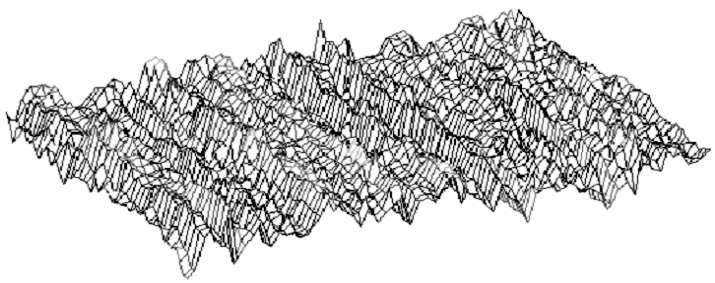


New experiments on roughness

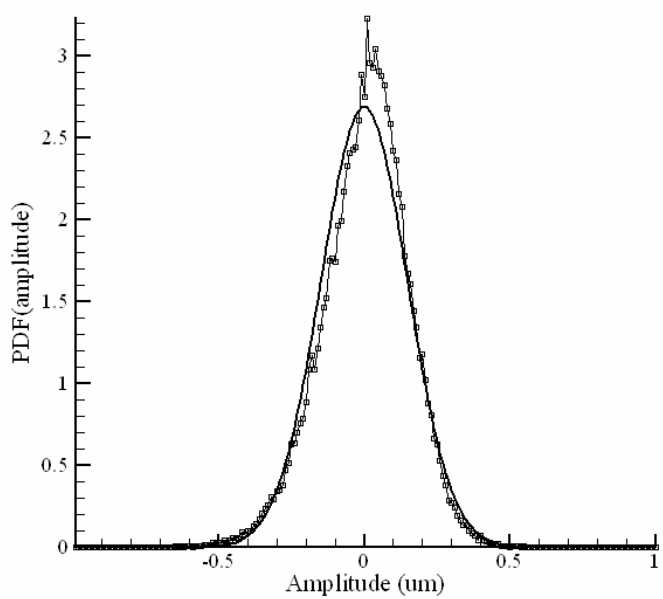
- Use Superpipe apparatus to study different roughness types by installing different rough pipes
- Advantage: able to cover regime from smooth to fully rough with one pipe
- Honed surface roughness
  - To help establish where Superpipe data becomes rough (10×10^6, or 28×10^6, or 35×10^6, or what?)
  - To help characterize an important roughness type (honed and polished finish) ($k_s = 6k_{rms}$, $k_s = 3k_{rms}$?)
- Commercial steel pipe roughness
  - Most important surface for industrial applications

Honed surface finish

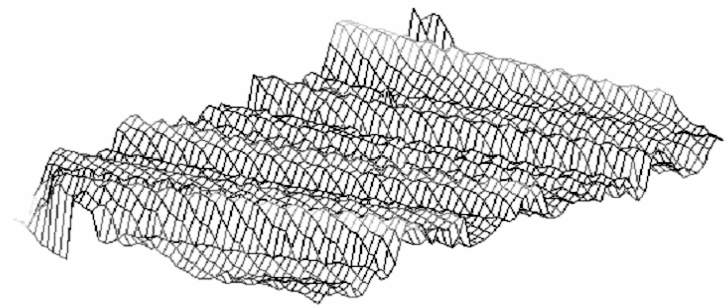
"Smooth" pipe, 6μin



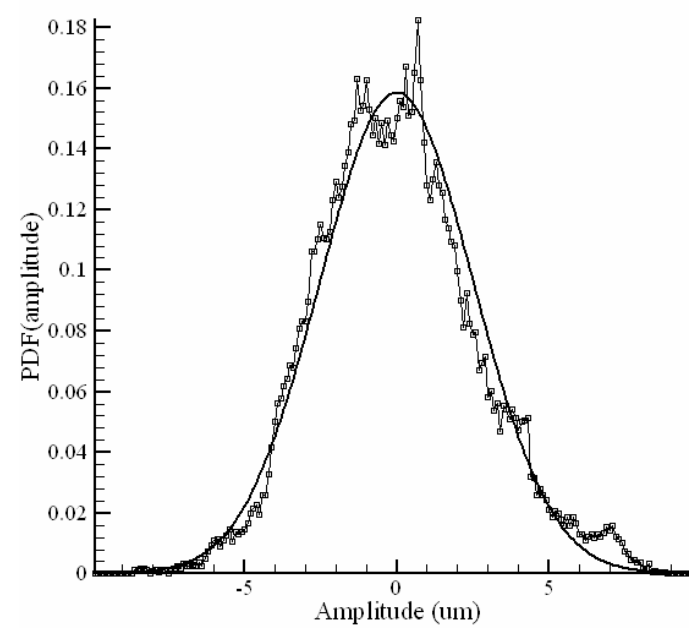
$$\frac{k_{RMS}}{D} = 1.16 \times 10^{-6} \quad k_{RMS} = 0.15\mu m$$



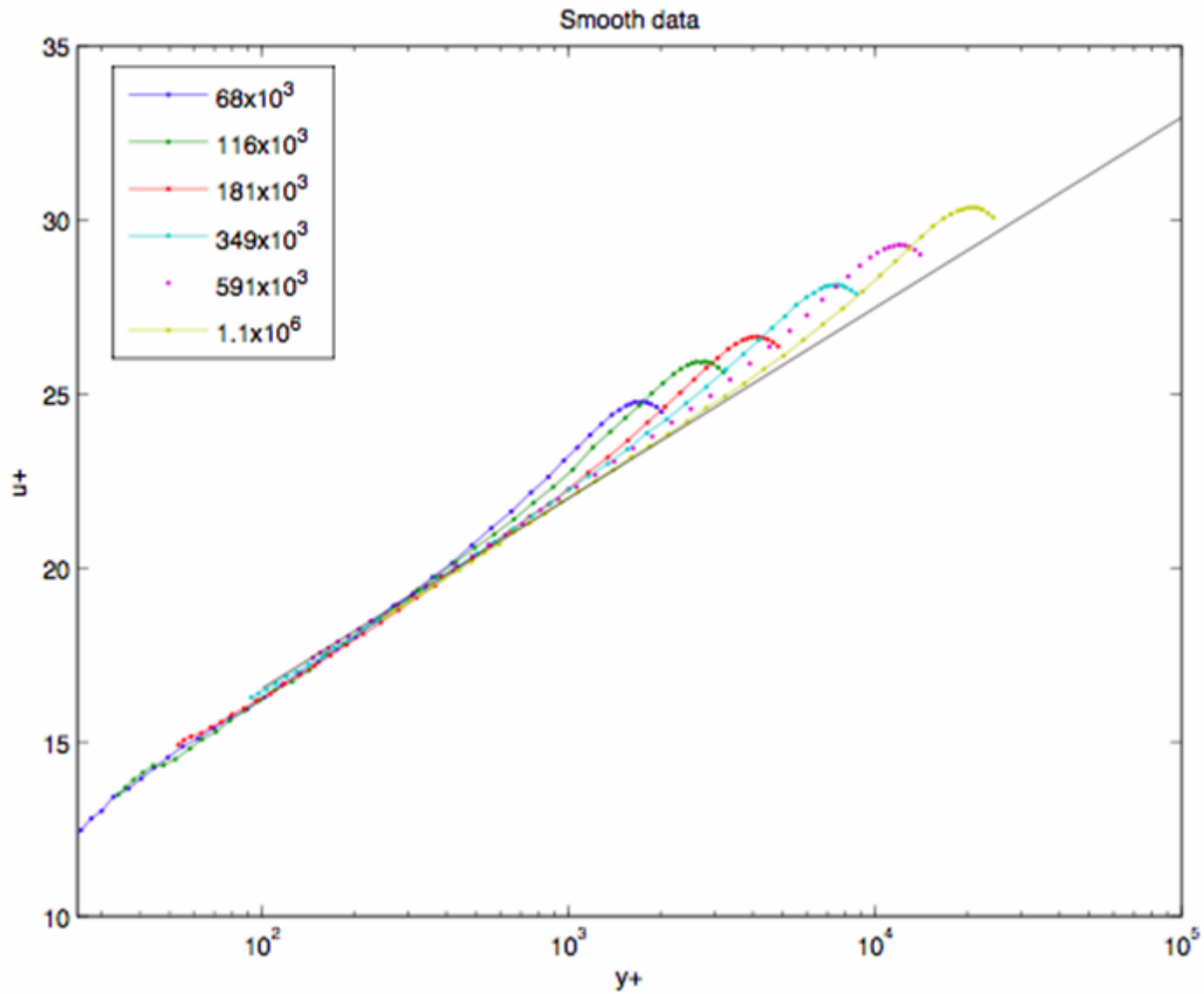
Honed rough pipe, 98μin



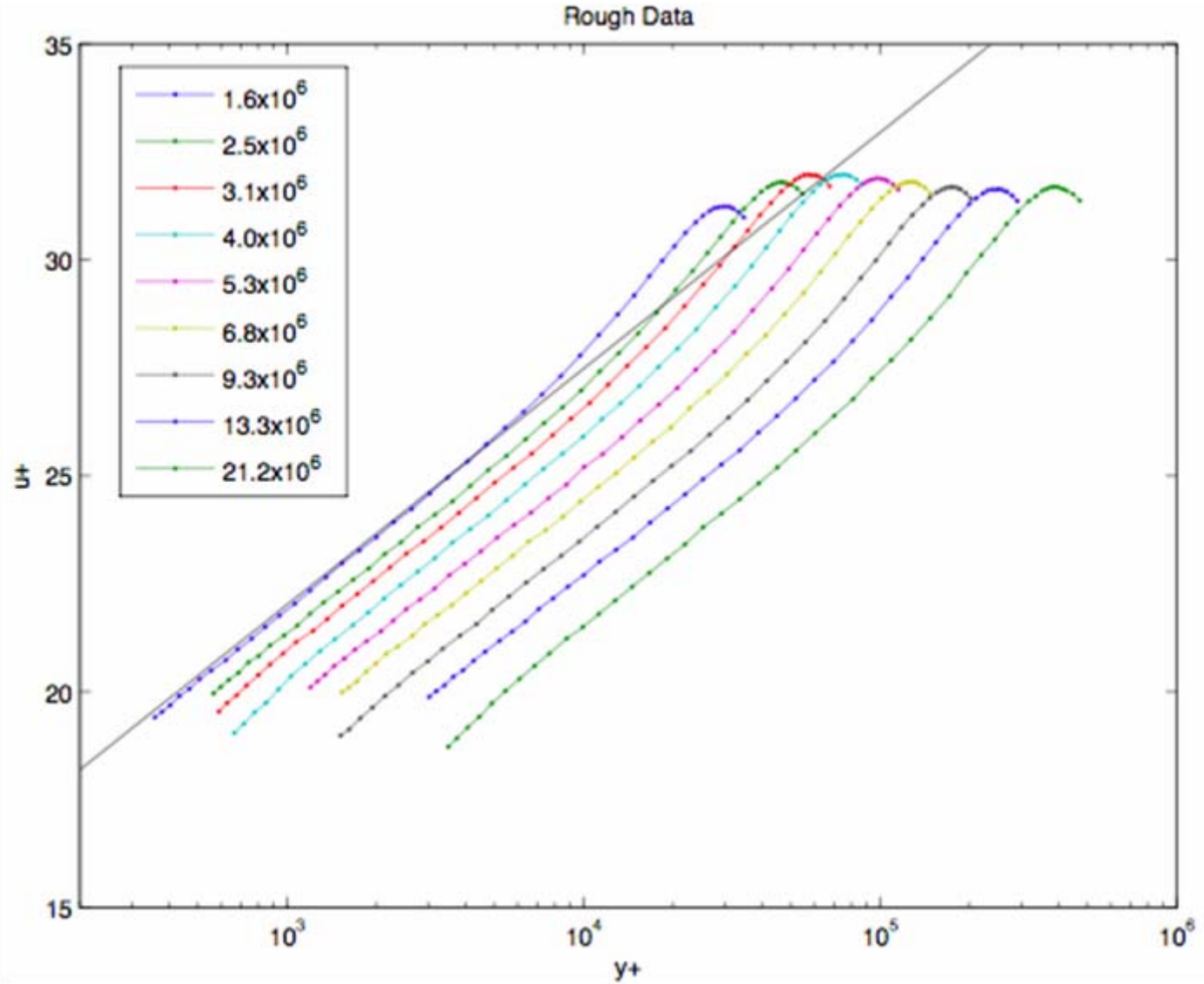
$$\frac{k_{RMS}}{D} = 1.94 \times 10^{-5} \quad k_{RMS} = 2.5\mu m$$



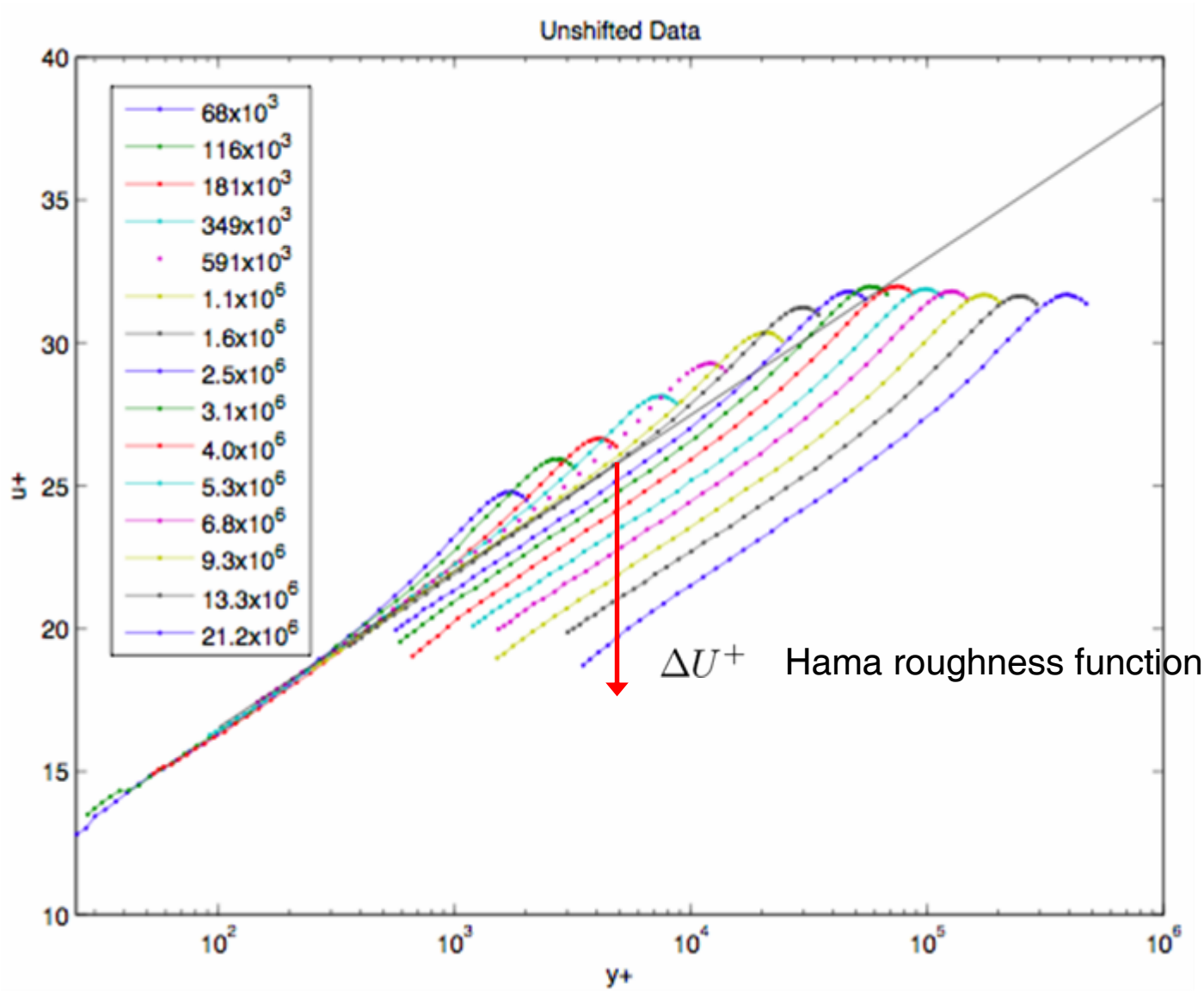
Results in smooth regime



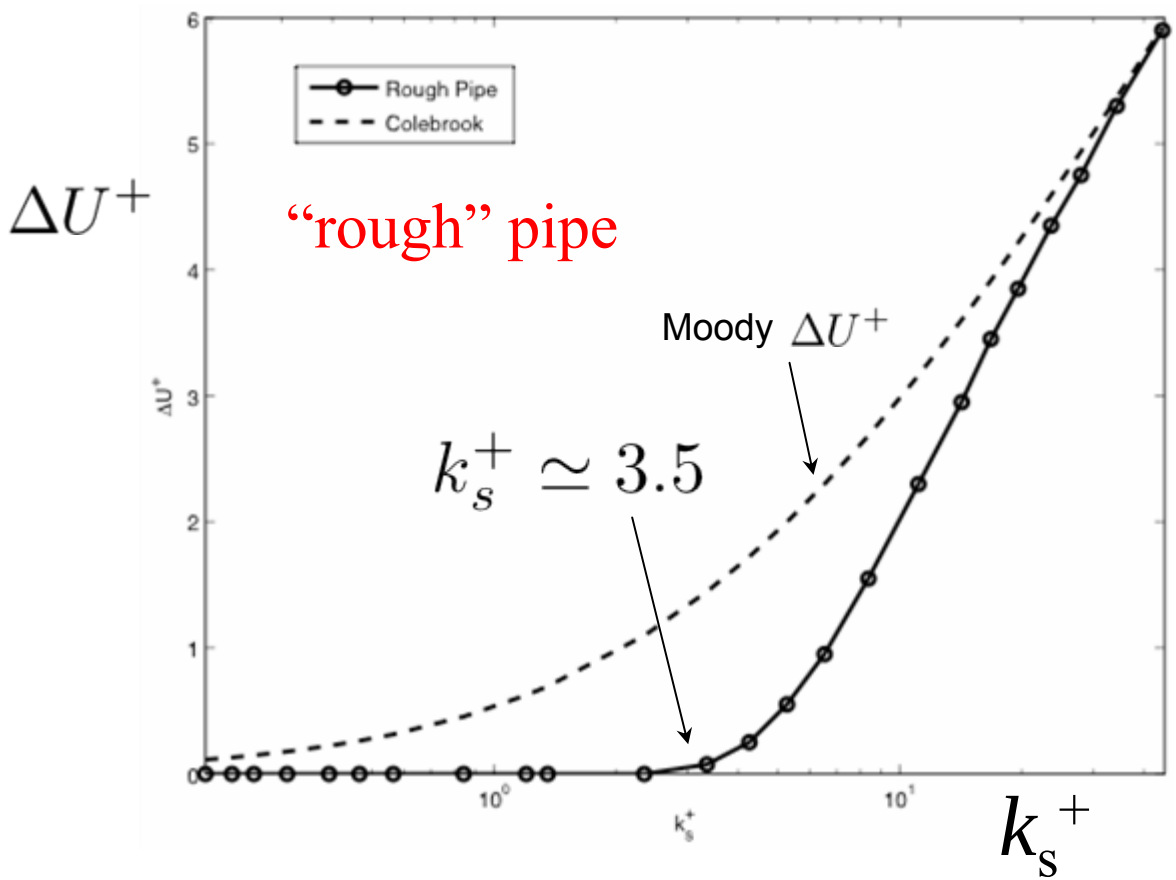
Results: transitional/rough regime



Inner scaling - all profiles

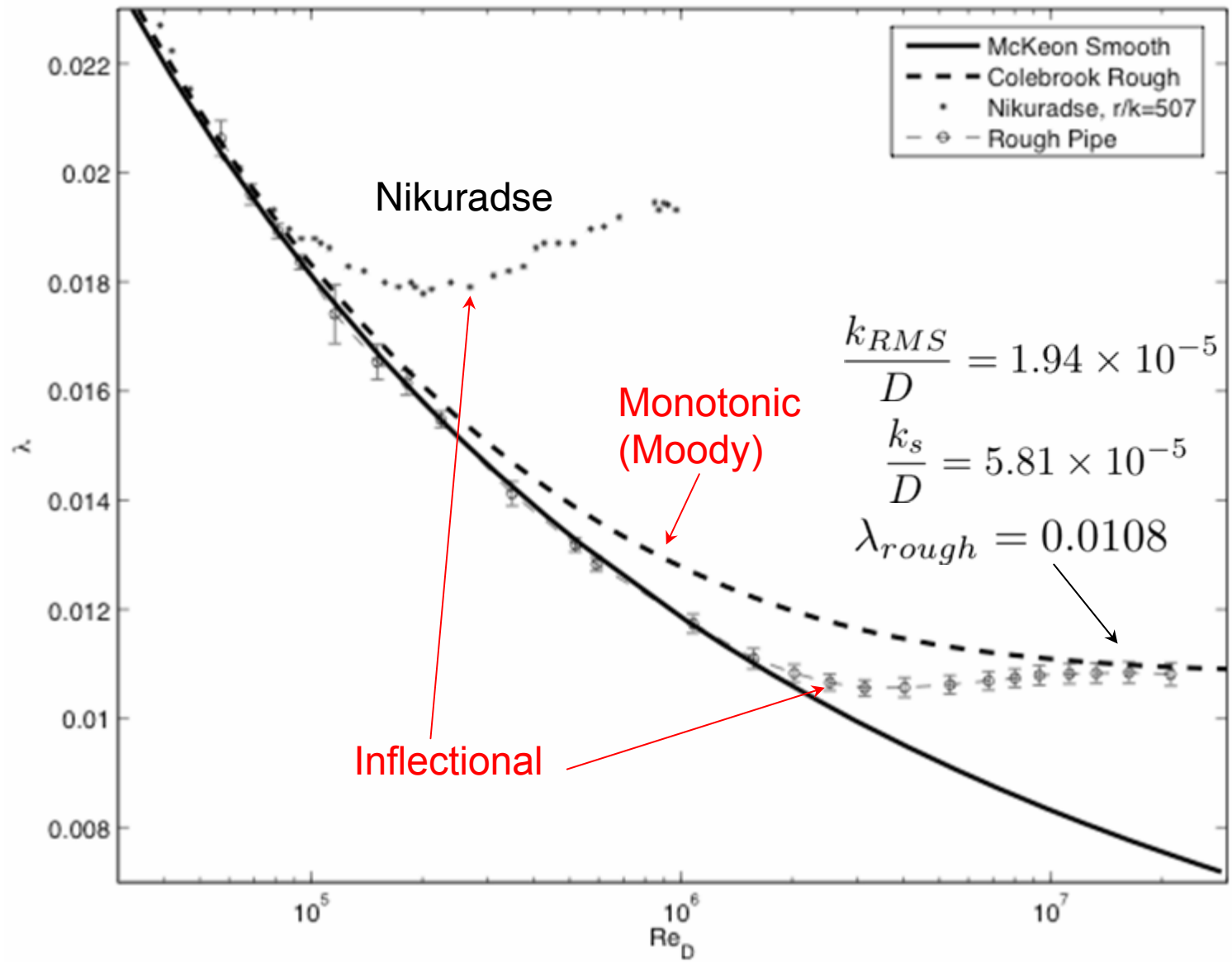


Rough pipe: Inner scaling

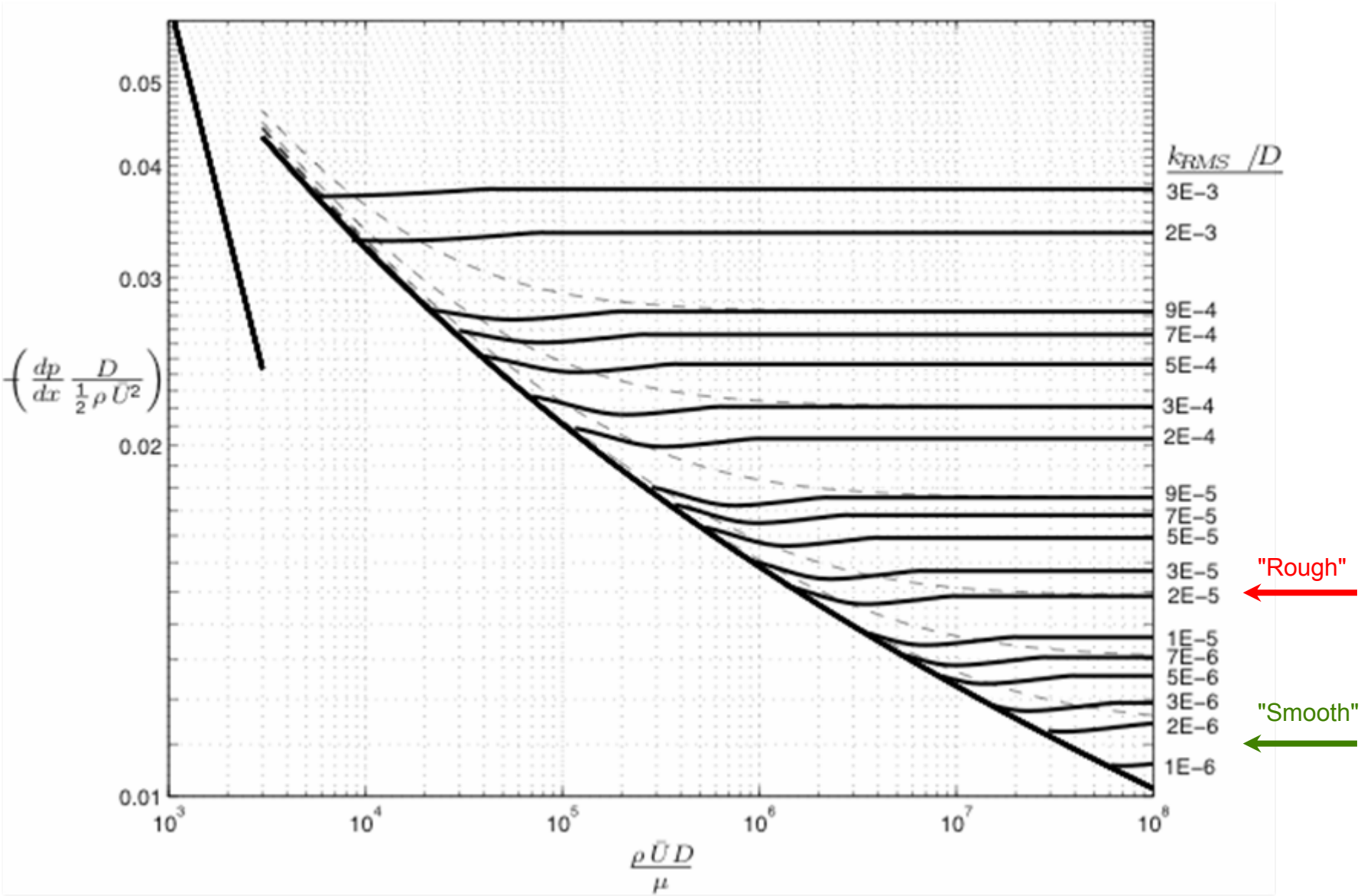


Therefore "smooth" Superpipe was smooth for Re_D </= 21 x 10^6

Friction factor results for rough pipe

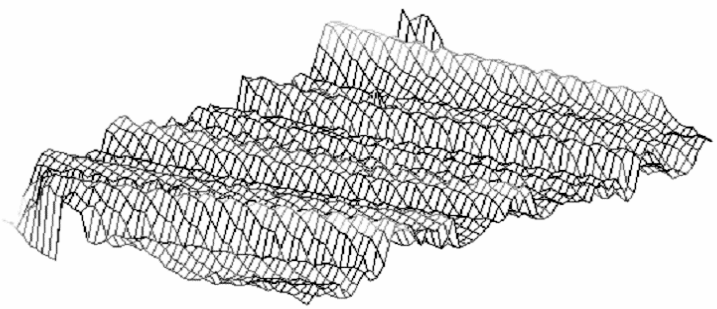


Revised resistance diagram for honed surfaces

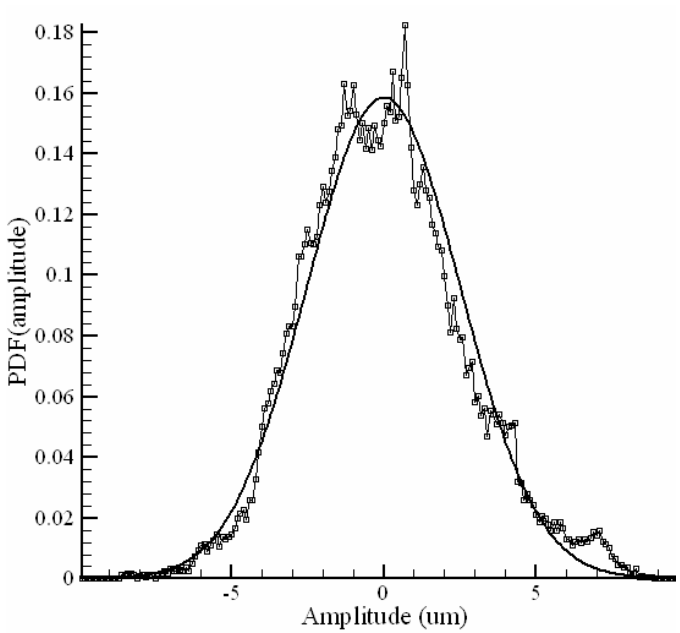


Commercial steel surface roughness

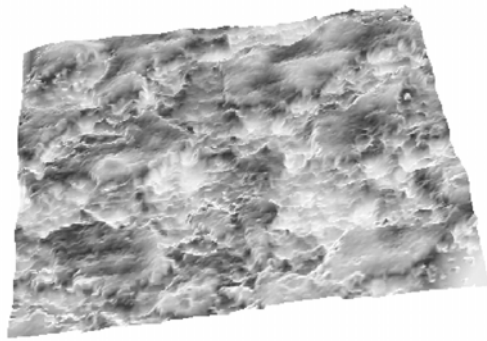
Honed rough pipe, 98µin



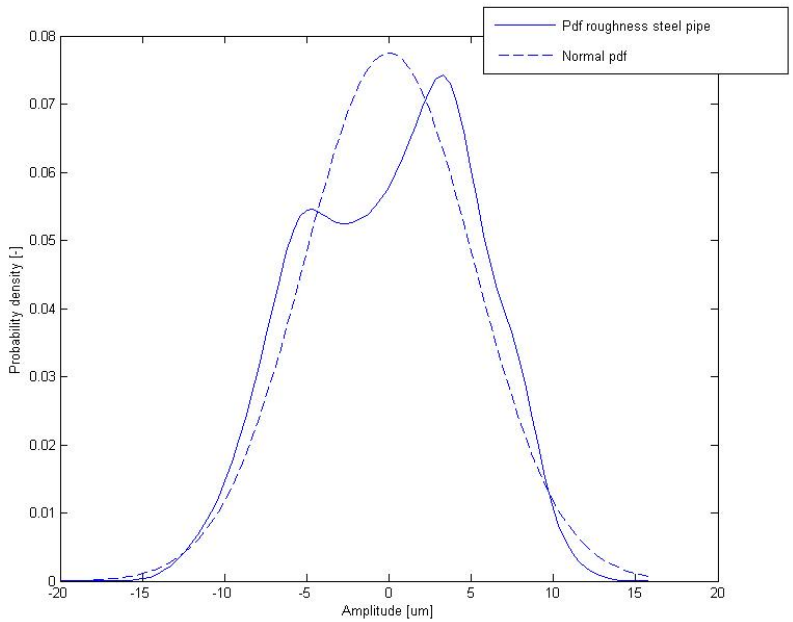
$$\frac{k_{RMS}}{D} = 1.94 \times 10^{-5} \quad k_{RMS} = 2.5\mu m$$



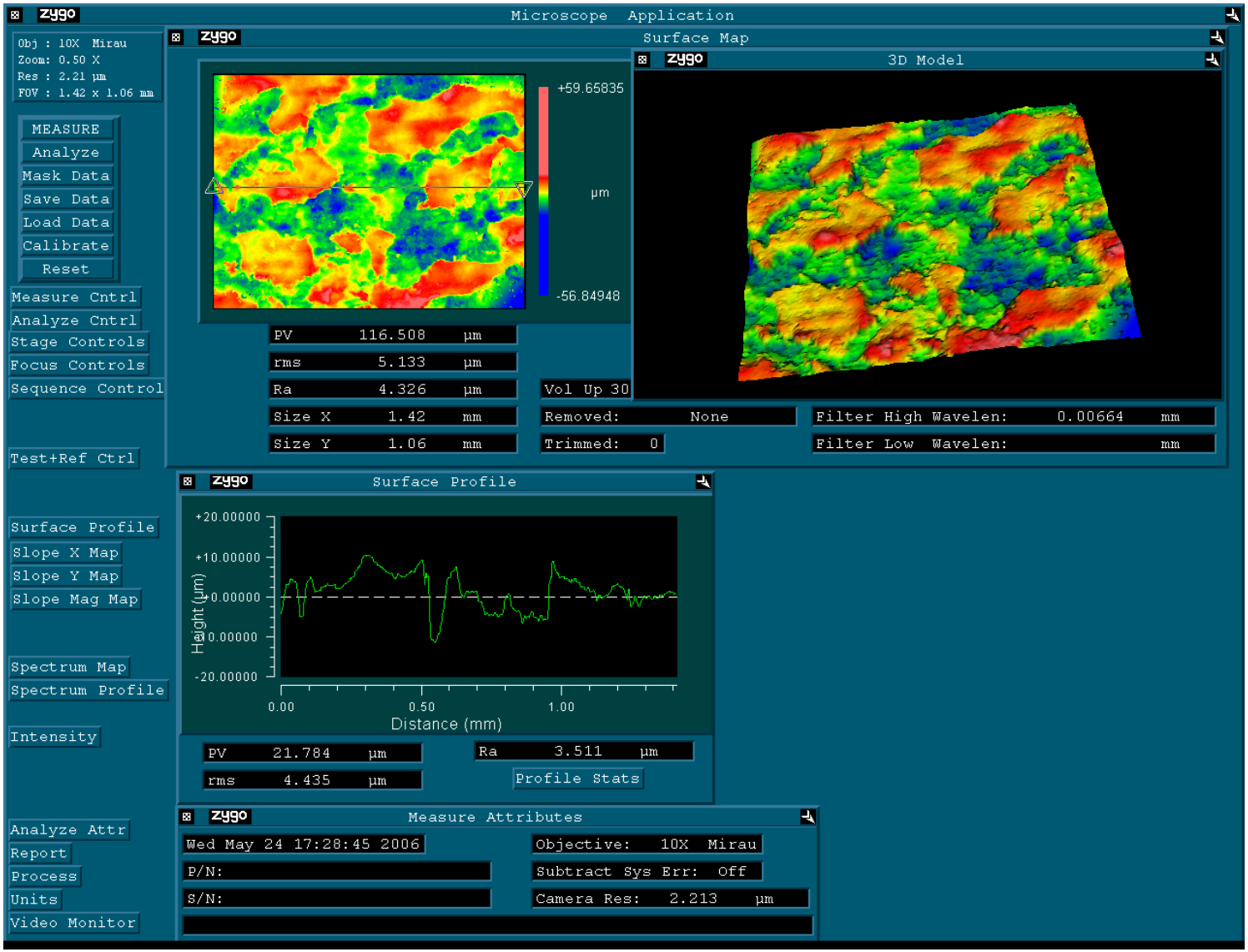
Commercial steel rough pipe, 195µin



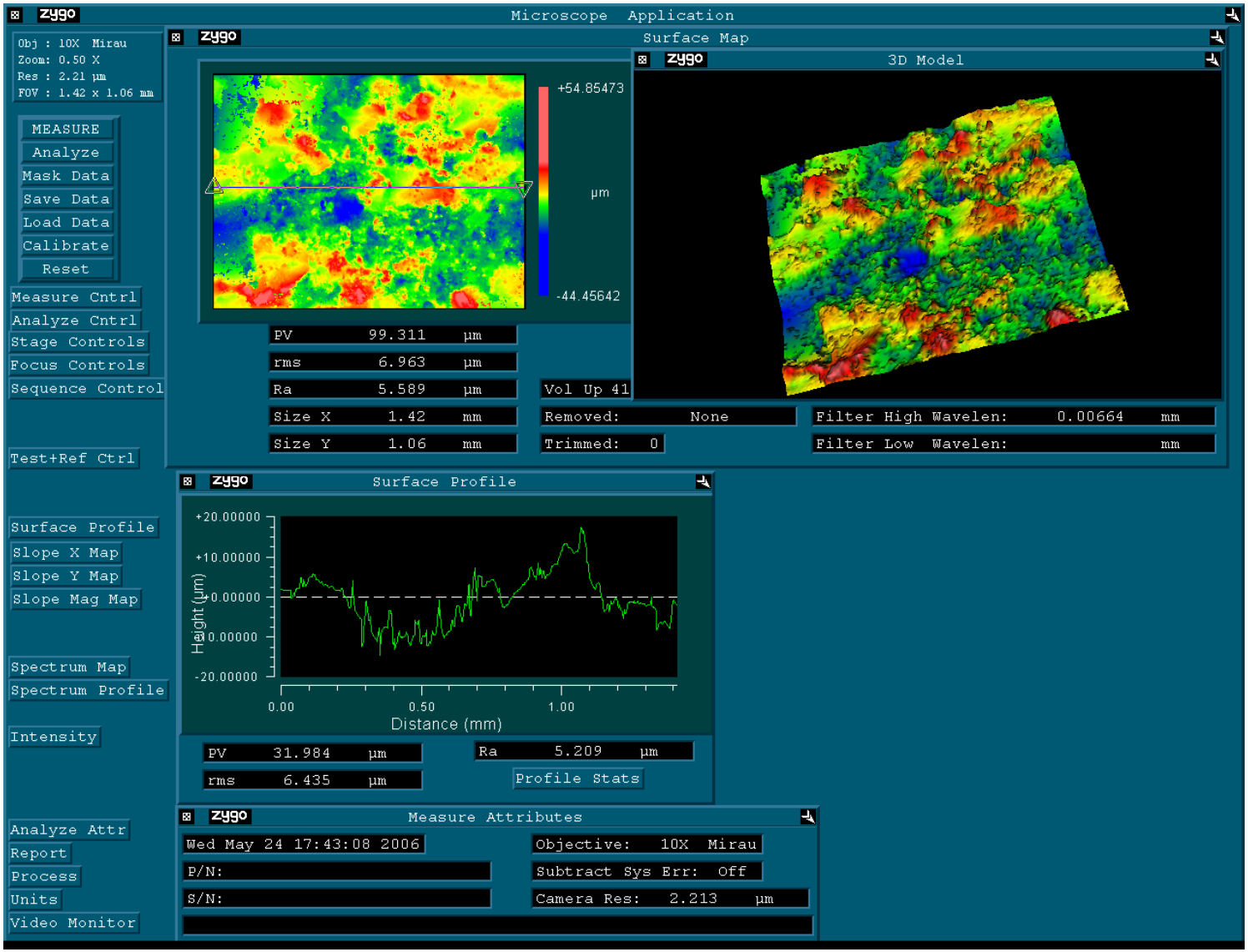
$$\frac{k_{RMS}}{D} = 3.82 \times 10^{-5} \quad k_{RMS} = 5.0\mu m$$



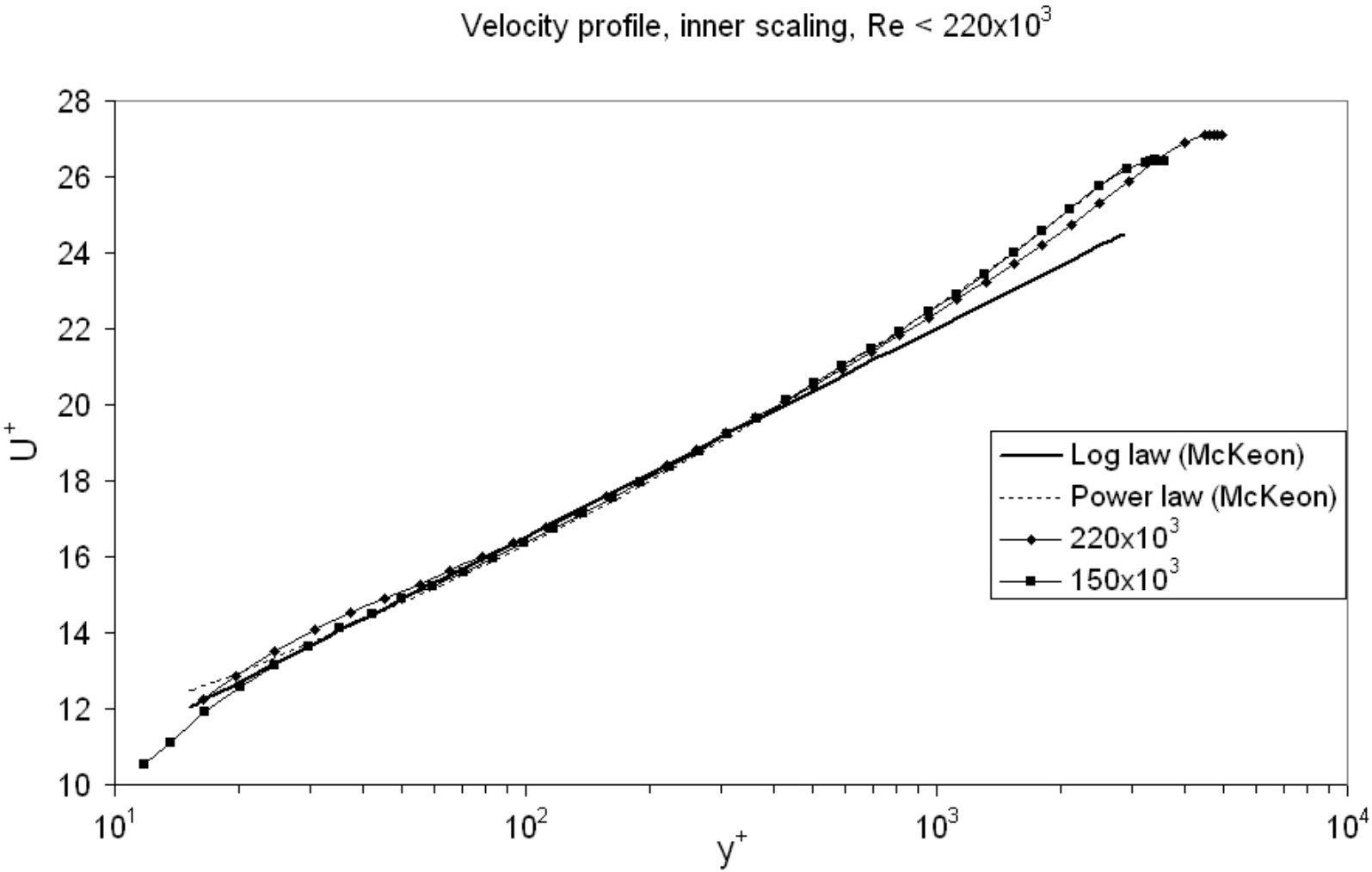
Sample 2: non-rust spot



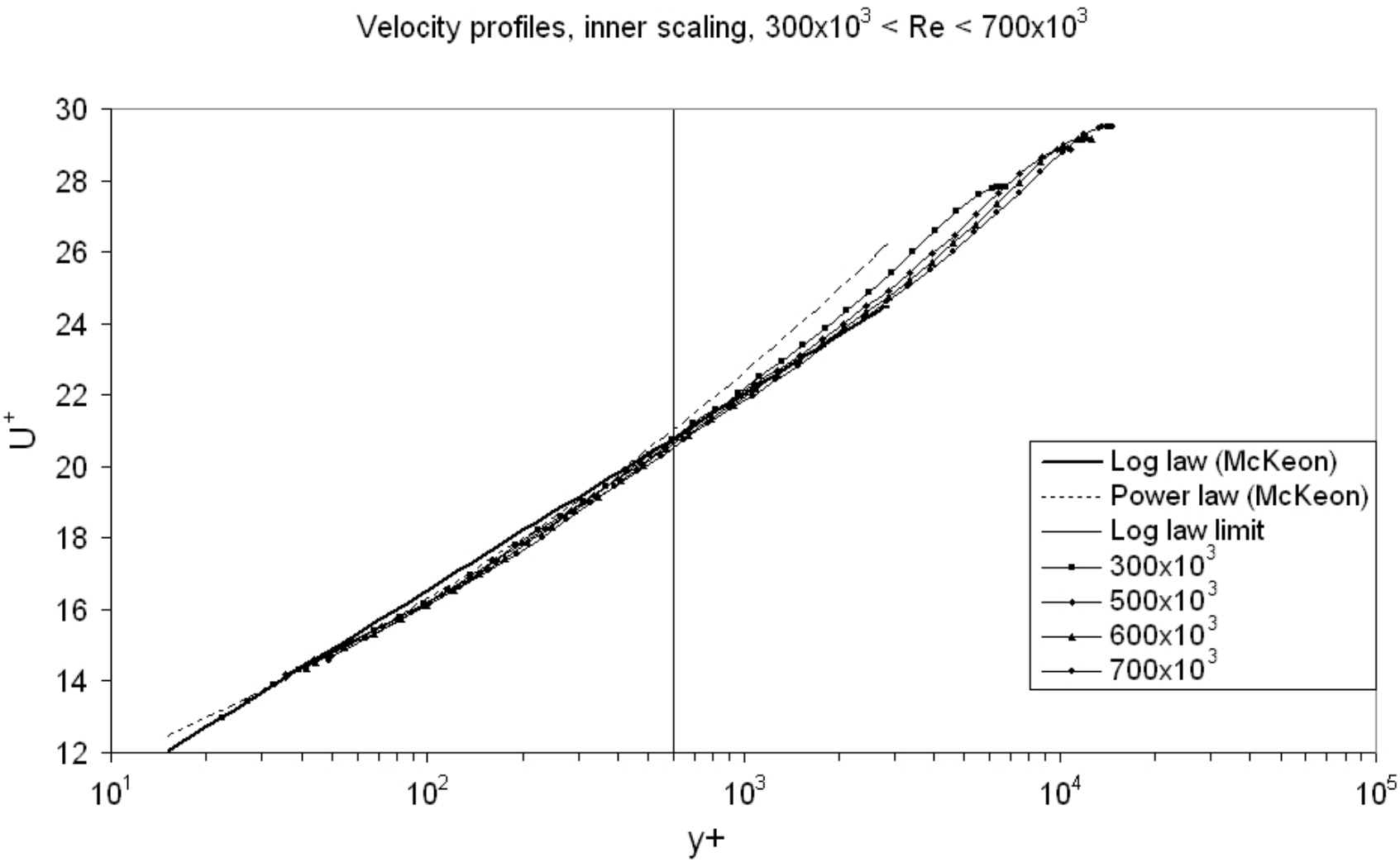
Sample 2: rust spot



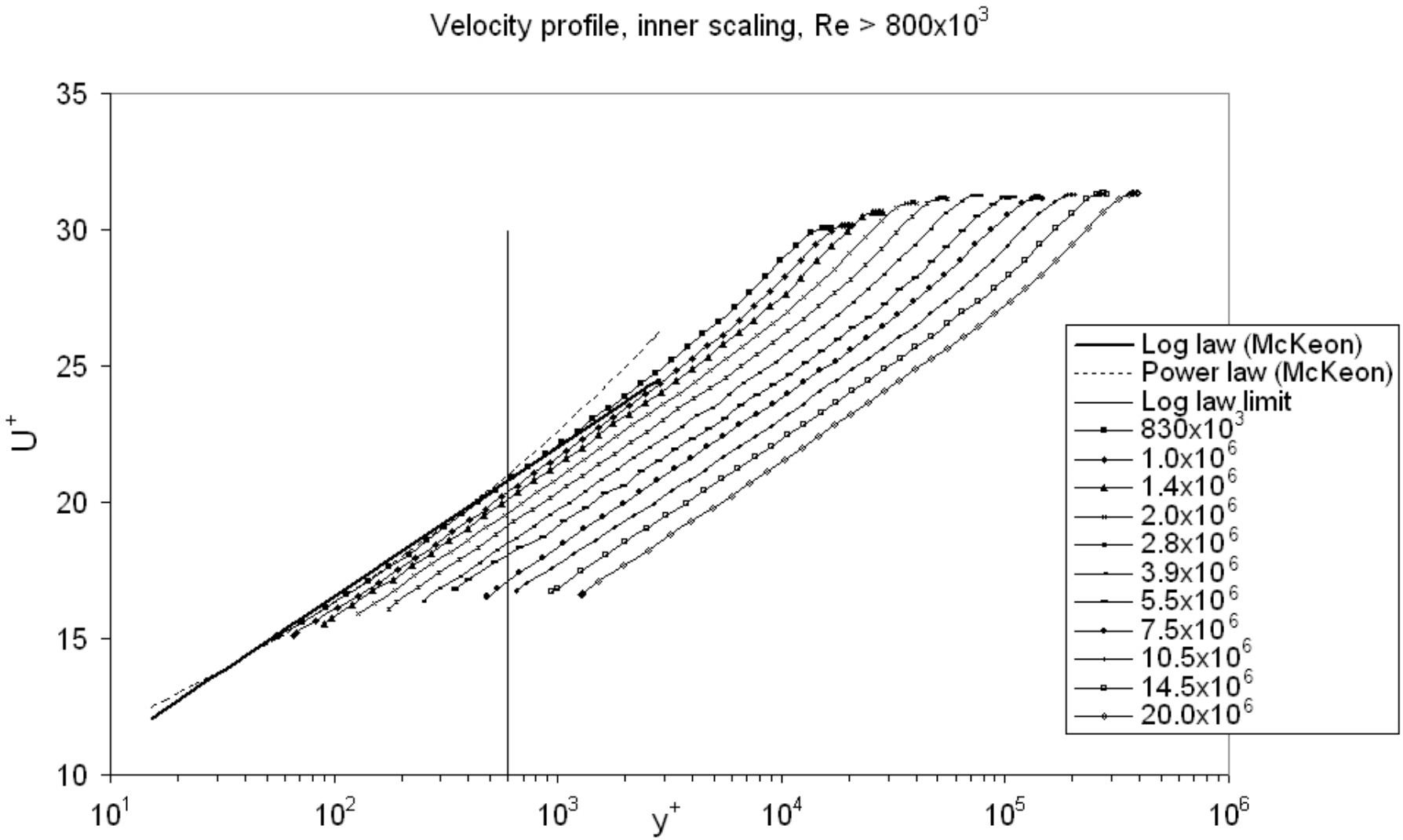
Velocity profiles: inner scaling



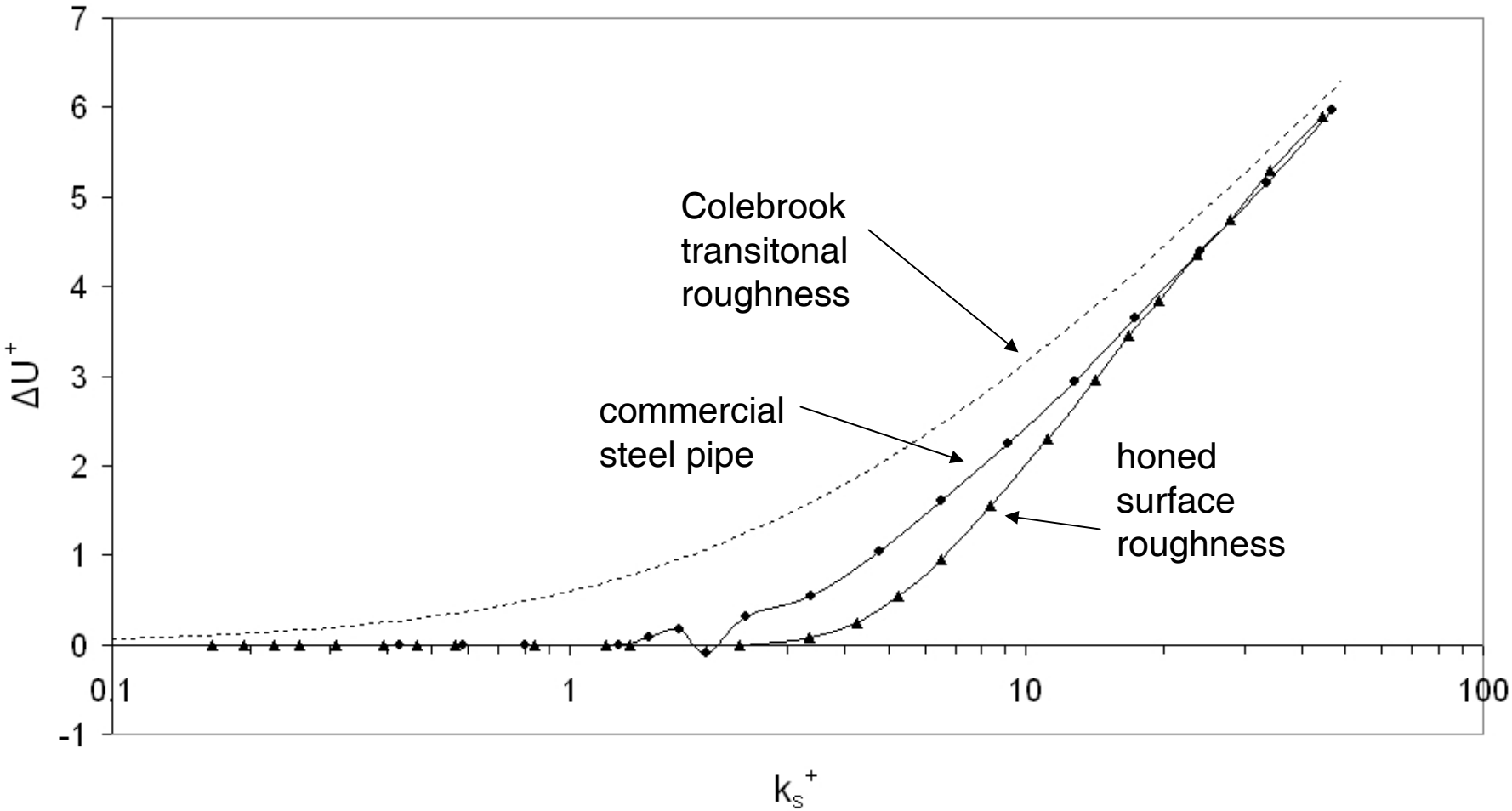
Velocity profiles: inner scaling



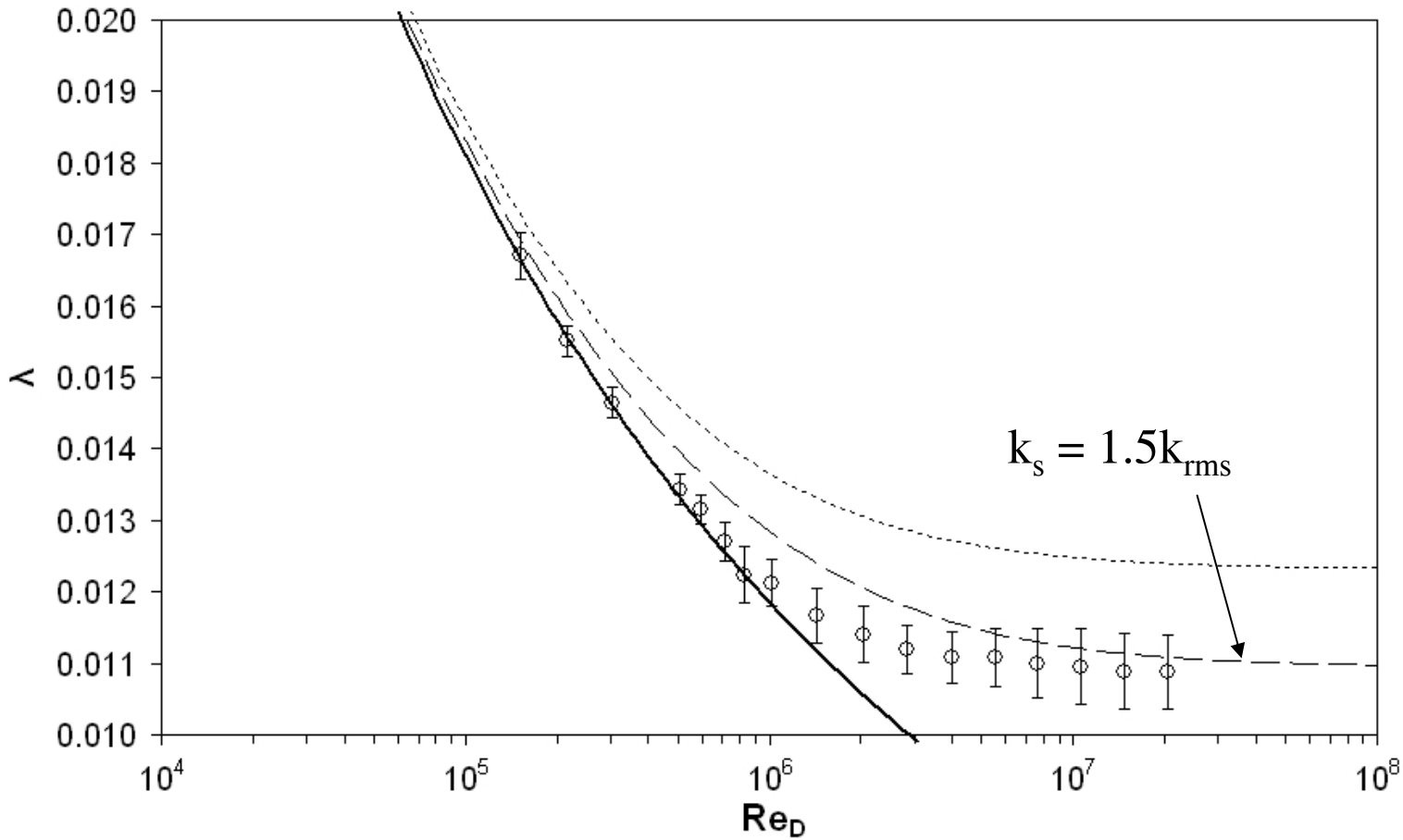
Velocity profiles: inner scaling



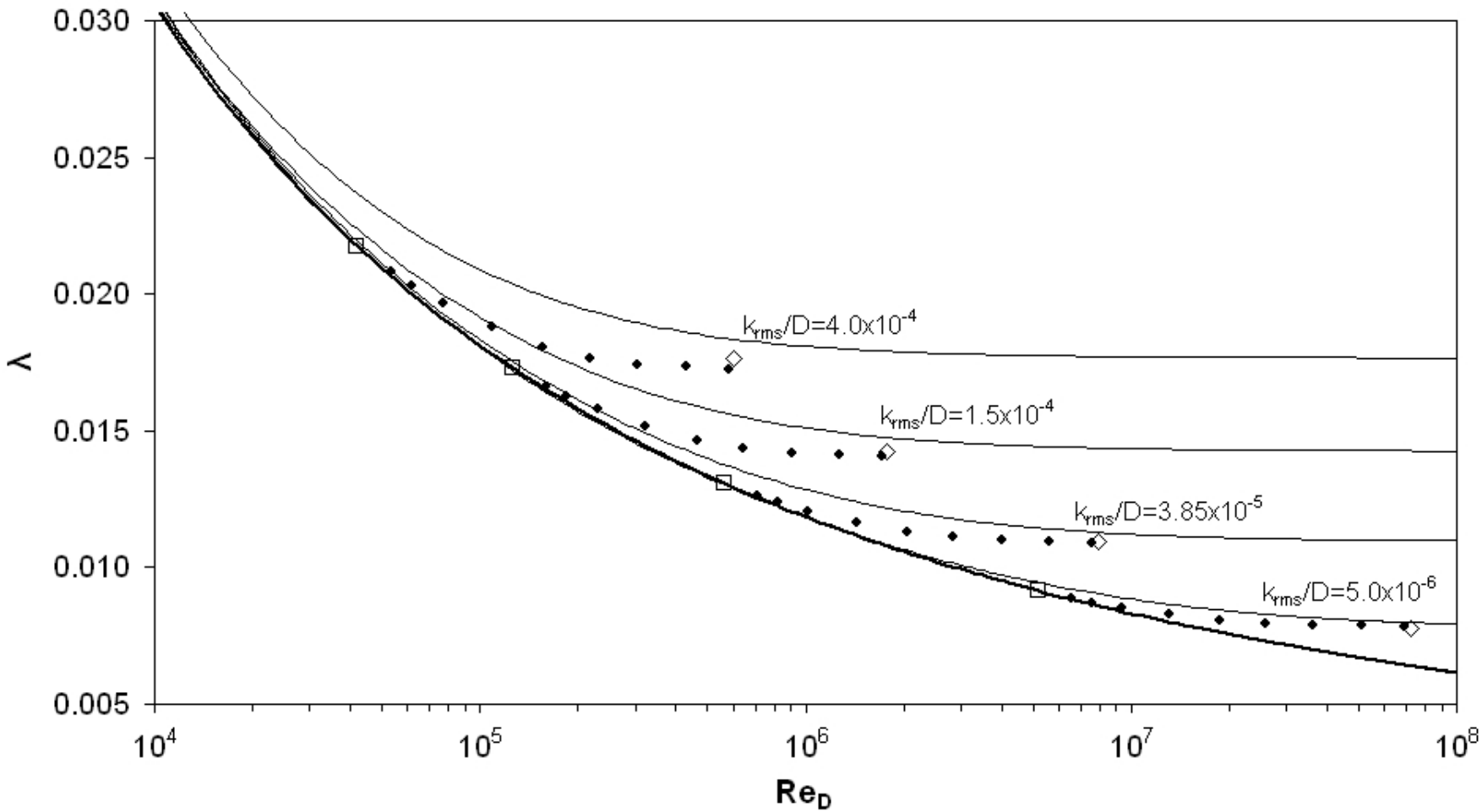
Hama roughness function



Commercial steel pipe friction factor



Moody diagram for commercial steel pipe



Rough pipe summary

- Honed surface roughness
- Smooth ⟶ transitional ⟶ fully rough
- $k_{rms}/D = 19 \times 10^{-6}$
- Pipe L/D = 200
- $Re_D = 57 \times 10^3$ to 21×10^6
- Smooth for $k_s^+ < 3.5$
- $k_s = 3.0k_{rms}$
- Inflectional friction factor not monotonic (Nikuradse not Colebrook)

- Commercial steel pipe roughness
- Smooth ⟶ transitional ⟶ fully rough
- $k_{rms}/D = 38 \times 10^{-6}$
- Pipe L/D = 200
- $Re_D = 93 \times 10^3$ to 20×10^6
- Smooth for $k_s^+ < 3.1$
- $k_s = 1.5k_{rms}$ (instead of $3.5k_{rms}$)!
- Friction factor monotonic (but not Colebrook)

Why the Moody Diagram needs updating

- Prandtl's universal friction factor relation is not universal (breaks down at higher Reynolds numbers: $>3 \times 10^6$)
- Transitional roughness regime is represented by Colebrook's transitional roughness function using an equivalent sandgrain roughness, which takes no account of individual roughness types
- Honed surfaces are inflectional not monotonic
- Commercial steel pipe monotonic but not Colebrook

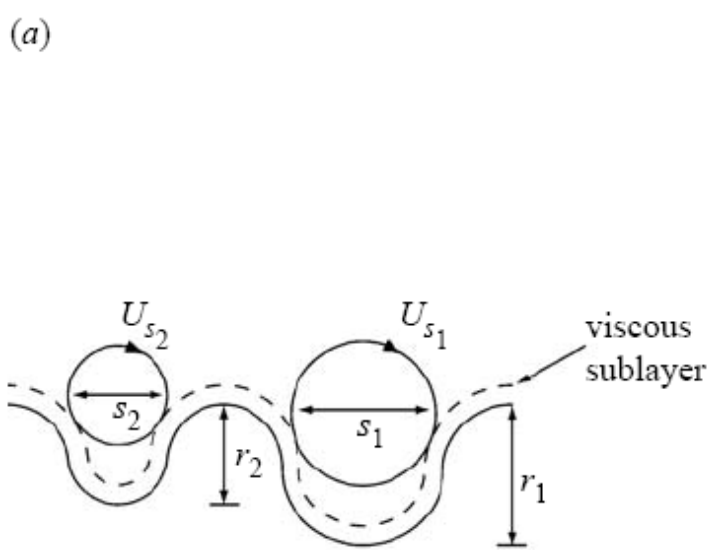
- The limitations of the Moody Diagram were well-known (e.g., Hama), but no match for text book orthodoxy

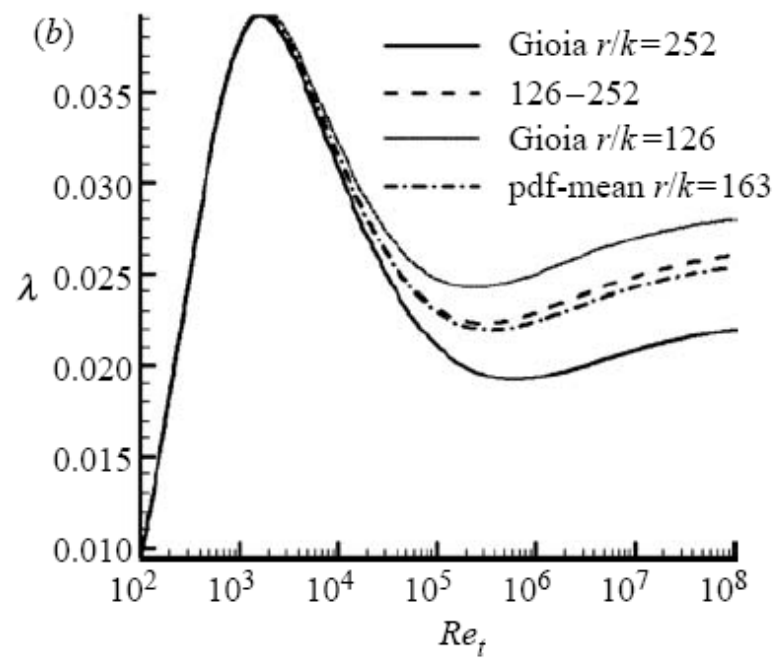
Where do we go from here?

- More experiments, more data analysis?
  - Schultz and Flack
- A predictive theory?
  - Gioia and Chakraborty
- Petascale computing?
  - Moser, Jimenez, Yeung

Goia and Chakraborty's (2006) model

- Model the energy spectrum in the inertial and dissipative ranges
- Use the energy spectrum to estimate the speed of eddies of size s
- Model the shear stress on roughness element of size s as $\tau \simeq \rho \bar{U} u_s$
- Hence $\lambda \propto u_s / \bar{U}$, then integrate across all scales to find λ

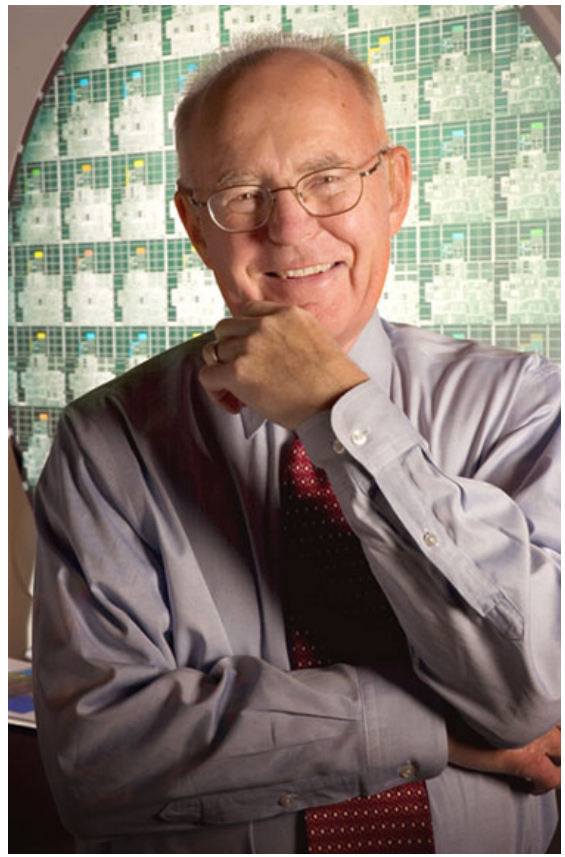




Prospects for Computation: Moore's Law

Intel co-founder Gordon Moore

April, 1965



2005

Petascale computing

- Earth Simulator (2004): 36 x 10^{12} flops peak
  - DNS of 4096^3 isotropic turbulence
- Petascale computing (2007): Blue Gene/P 3 x 10^{15} flops peak
  - Remarkable resource, but what questions can it answer?
- Example: DNS of channel flow
  - Bob Moser, UT Austin
  - $Re_\tau = u_\tau R/\nu$ (approx = $Re_D/40$)

Channel flow simulations

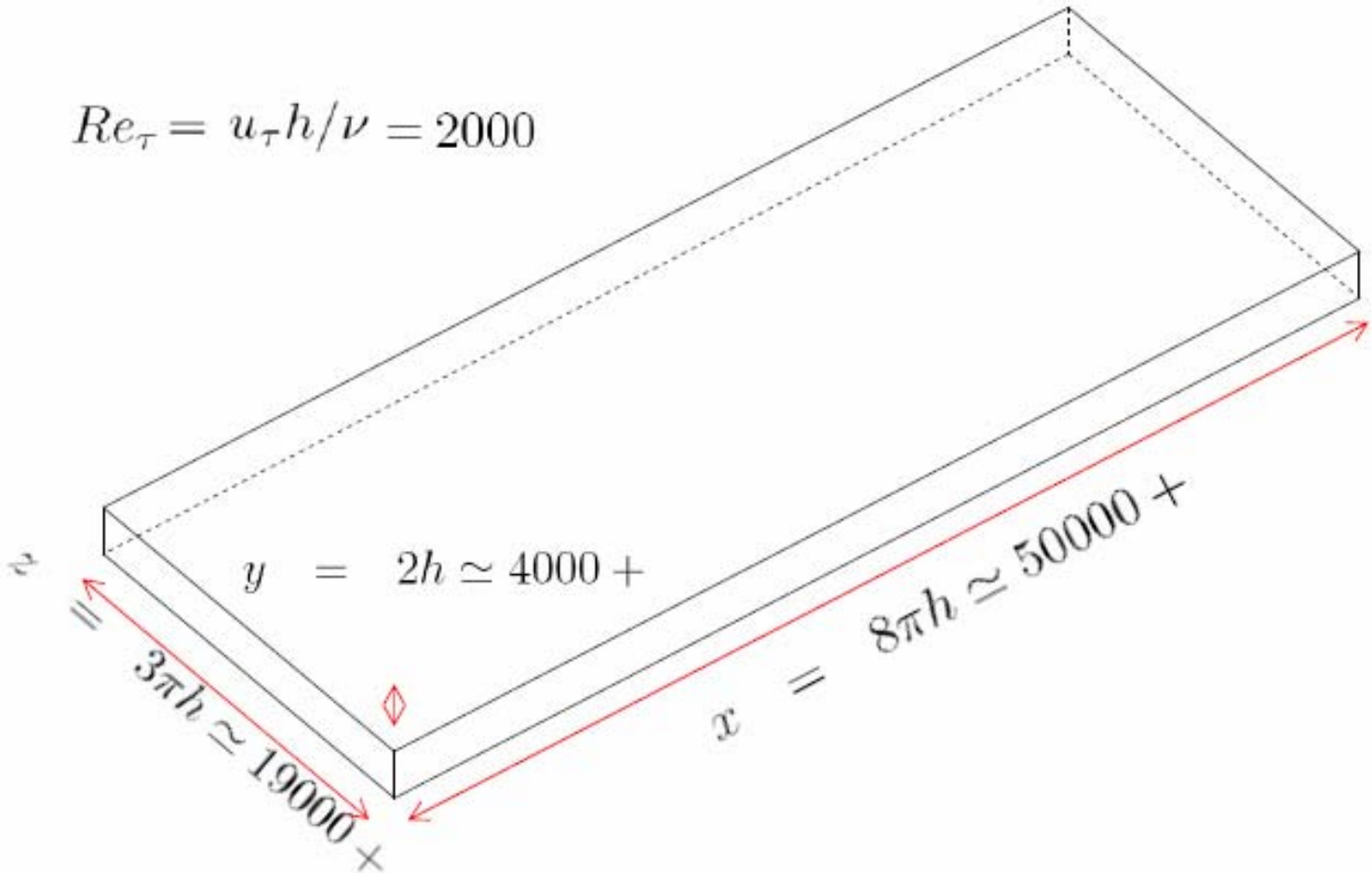
- Twin goals: High Reynolds number, Large spatial domain

Case	Re_τ	L_x/h	L_z/h	TU_b/L_x	N_x	N_z	N_y	$N_t/10^5$
L550	547	8π	4π	10	1536	1536	257	1.1
L950	934	8π	3π	10	3072	2304	385	2.1
S1900	1880	π	$\pi/2$	40	768	768	769	0.5
L2000	2003	8π	3π	10	6144	4608	633	4

del Álamo, Jiménez, Zandonade & Moser (2004) *J. Fluid Mech.* **500**, 135-144.

Hoyas & Jimenez (2006) *Phys. of Fluids* **18**, 011702.

Domain L2000



Resources for L2000 (Re_D approx 80,000)

- Processors: 2048 (Power 4)
- Memory: 875 GB ($\sim$ 430 MB/processor)
- Restart File Size $\sim$ 64GB (2 DOF/mode)
- Total simulation time: $\sim$ 122 days, or 6×10^6 proc. hrs

How much higher can we go?
How much higher need we go?

Extended log-law (Moser)

- To this level of approximation:

$$\left.\frac{\partial u^+}{\partial y^+}\right|_i \approx \frac{1}{\kappa y^+} + \epsilon\alpha + \frac{\epsilon\beta}{y^+} \qquad \text{for } y^+ \gg 1$$

$$\left.\frac{\partial u^+}{\partial \tilde{y}}\right|_o \approx \frac{1}{\kappa \tilde{y}} + \alpha + \frac{\epsilon\beta}{\tilde{y}} \qquad \text{for } \tilde{y} \ll 1$$

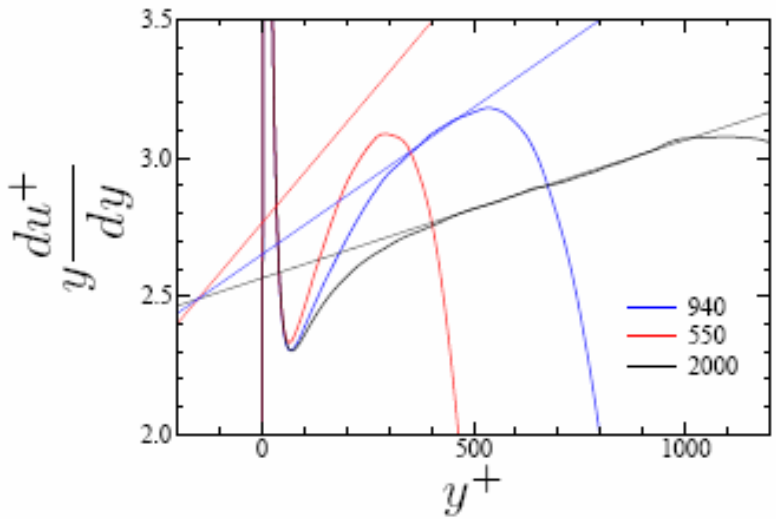
- Integrate to obtain:

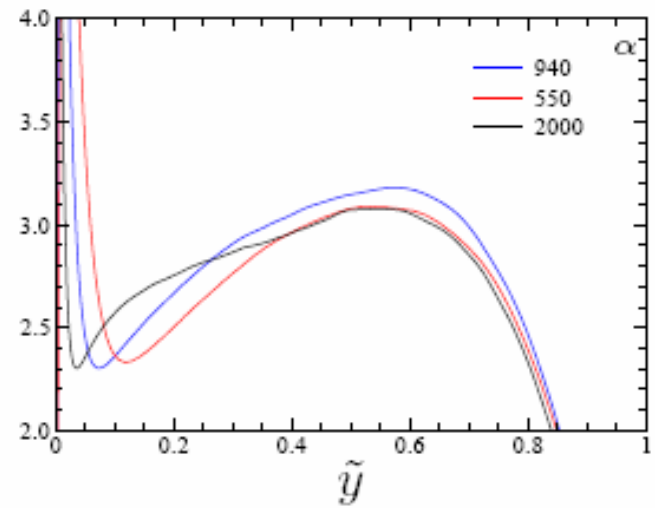
$$u_i^+ = \left(\frac{1}{\kappa} + \frac{\beta}{Re_\tau}\right) \ln y^+ + \frac{\alpha y^+}{Re_\tau} + B_i$$

$$u_o^+ - u_c^+ = \left(\frac{1}{\kappa} + \frac{\beta}{Re_\tau}\right) \ln \tilde{y} + \alpha\tilde{y} + B_o$$

Comparison with DNS channel data

$$u_i^+ = \left(\frac{1}{\kappa} + \frac{\beta}{Re_\tau}\right)\ln y^+ + \frac{\alpha y^+}{Re_\tau} + B_i \qquad u_o^+ - u_c^+ = \left(\frac{1}{\kappa} + \frac{\beta}{Re_\tau}\right)\ln \tilde{y} + \alpha\tilde{y} + B_o$$





$$\alpha \approx 1.0 \qquad \kappa \approx 0.4 \qquad \beta \approx= 150$$

- Reasonable approximation for $y^+ > 300$ and $\tilde{y} < 0.45$
- First convincing overlap at $Re_\tau = 2000$, need $Re_\tau > 4000$

From Re_τ = 2000 to 5000

- Just high enough to start seeing the asymptotic approach to $Re \to \infty$
  - "Log Layer"
  - Inner and outer scale separation
  - 2-D version of $1/k_x$ spectrum
- $Re_\tau \approx 5000$ will be high enough to confirm asymptote and be convincing
- $L_x \approx 40\delta$ will be a large enough domain to capture full range of scales

Resource requirements

- Computer Time (Peta Flop Hours – PFH)

$$PFH = C_t \tilde{L}_t \tilde{L}_x^2 \tilde{L}_z Re_\tau^4 \qquad C_t = 1.6 \times 10^{-16}$$

- Storage (Terabytes – TB)

$$TB = C_s \tilde{L}_x \tilde{L}_z Re_\tau^3 \qquad C_s = 4.3 \times 10^{-13}$$

Re_τ	$\tilde{L}_x$	$\tilde{L}_z$	$\tilde{L}_t$	PFH	ES Hrs	TB
5000	16	6	10	1500	96,000	2.64
5000	8	3	10	193	12,000	0.66
2000	8	3	10	5	300	0.08
2000	4	2	10	0.8	50	0.03
1000	8	3	10	0.3	19	0.01

$$\tilde{L}_x = L_x/\pi \qquad \tilde{L}_z = L_z/\pi \qquad \tilde{L}_t = L_t U_b / L_x$$

- With a sustained Peta-Flop, log-Layer is within reach

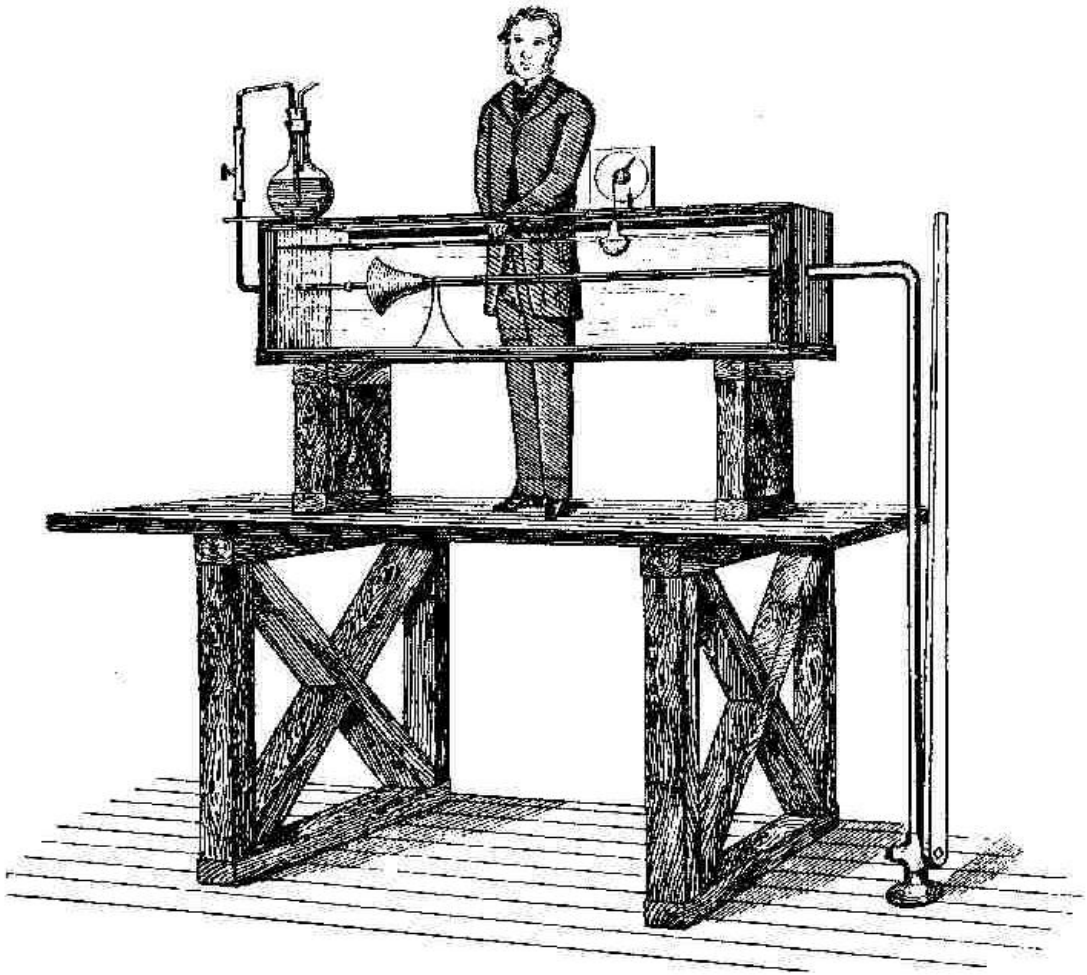
Are we done with channels at $Re_\tau = 5000$?

- Will give about an octave of log-law
- Will display "true" inner and outer regions
- Inadequate for high Reynolds number scaling (need $Re_\tau > 50{,}000$)
- What about roughness?
- With a 10 Petaflop machine
  - $Re_\tau = 5000$ is cheap enough to do experiments
  - Roughness studies?
- Maybe we can do roughness with a teraflop machine (if we are clever)

Conclusions

- Moody diagram should be revised, or used with caution
- Colebrook is pessimistic (makes us look good)
- Transitional roughness behavior not universal: depends on roughness
- Gioia model combined with better surface characterization may lead to predictive theory
- Petascale computing will provide powerful resource for fluids engineering, but maybe we'll "solve" roughness without it
- A "Golden Age" in the study of wall-bounded turbulence?

Questions??



Osborne Reynolds