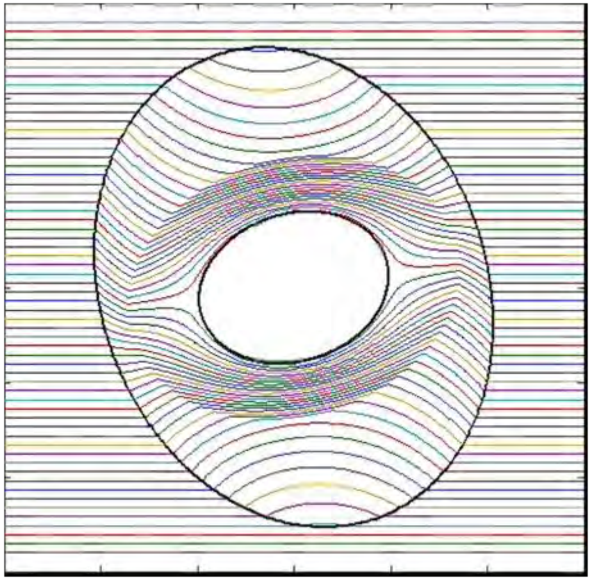
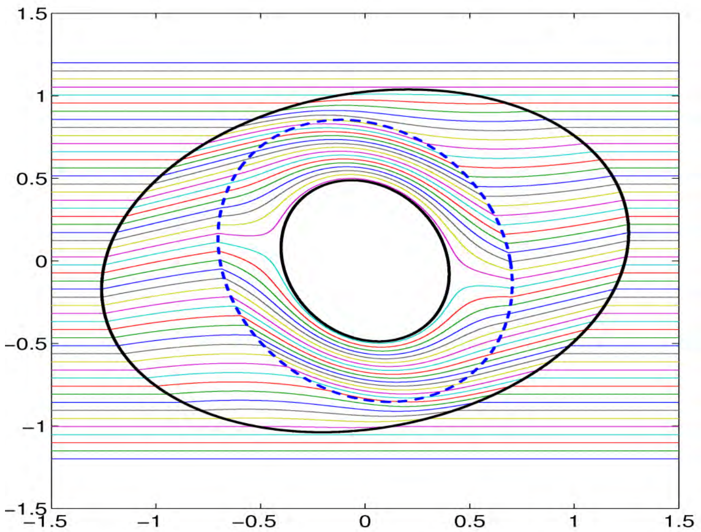
Rayleigh Lecture

Metamaterials in Acoustics and Vibration: From Theory to Practice

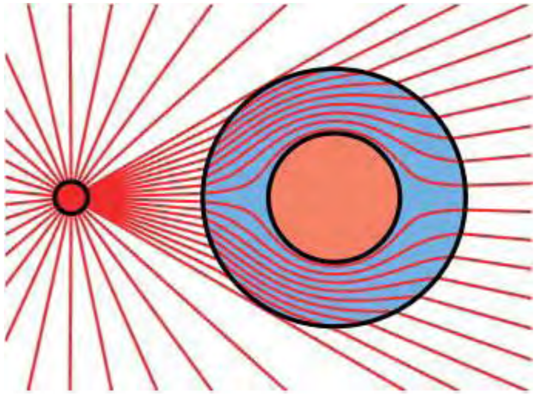


Andrew Norris
Rutgers University



ASME NCAD IMECE 2011

concepts



geometry = material properties

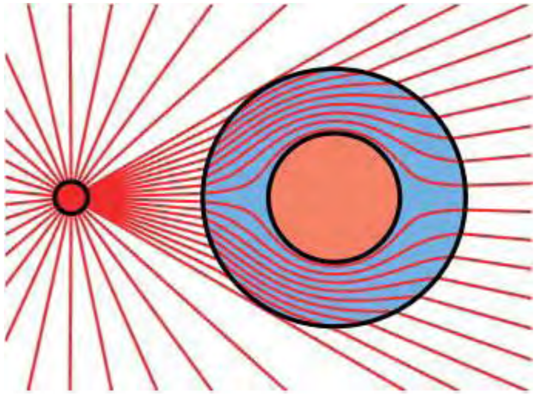
via homogenization or transformation or both

Applications:

narrow band : phononic crystals, negative index materials

broadband: acoustic cloaking

overview



Introduction: metamaterials

Phononic crystals – engineering the band gap
- waves in periodic systems
- examples of phononic crystals

Acoustic cloaking – engineering the impossible
- transformation acoustics
- 1D, 2D, cylindrical
- inertial materials
- Pentamode materials
- cloaking elastic waves

Metatheory for metamaterials?

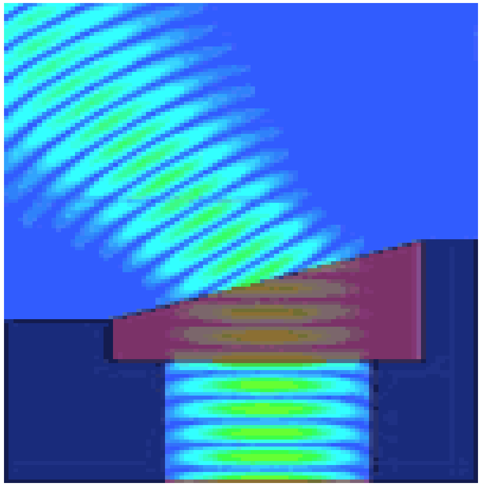
introduction

wiki/**metamaterial**

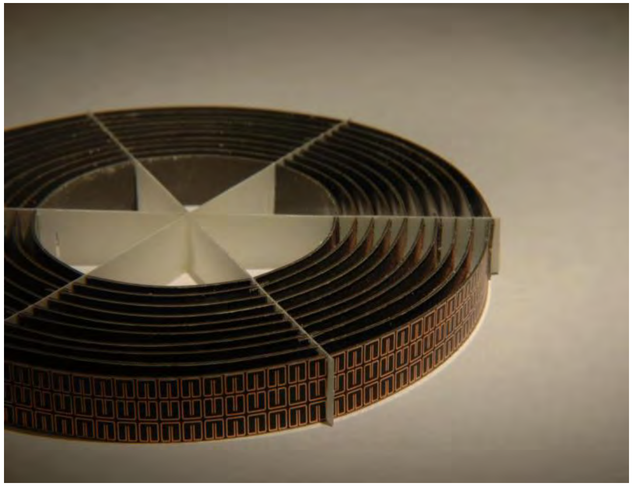
Word first used in 1999 by Rodger Walser of the University of Texas, Austin.

Metamaterials are:

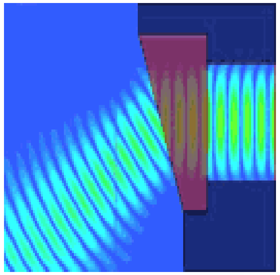
> *Macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation.*



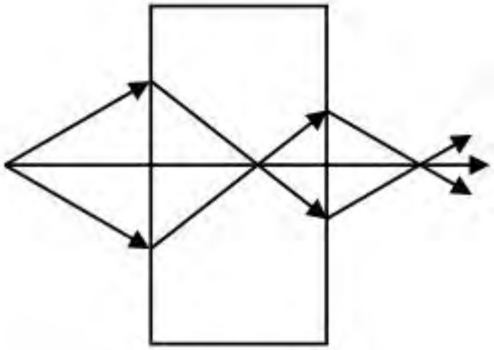
negative index of refraction



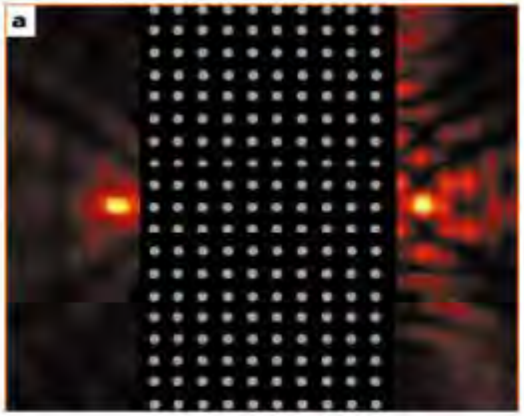
microwave **cloak**



Negative Refraction Makes a Perfect Lens



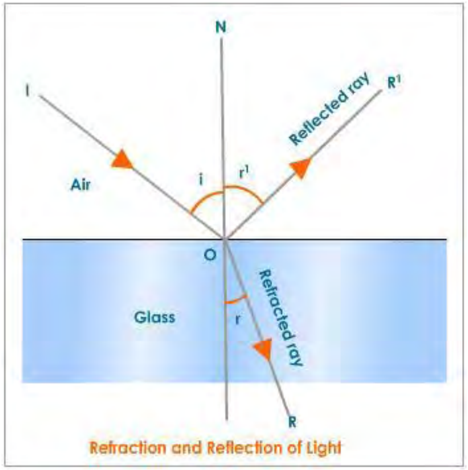
(Pendry, 2000)

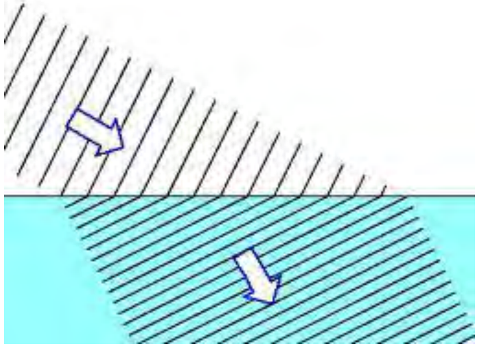


Parimi, Lu, Vodo and Sridhar
Nature 2003

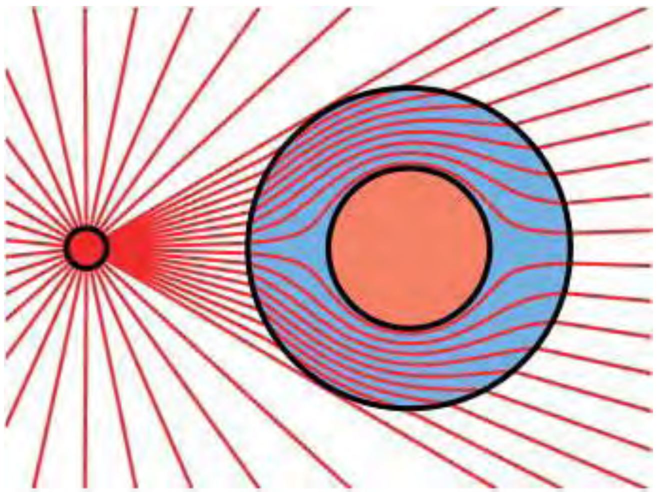
Microwave flat lens

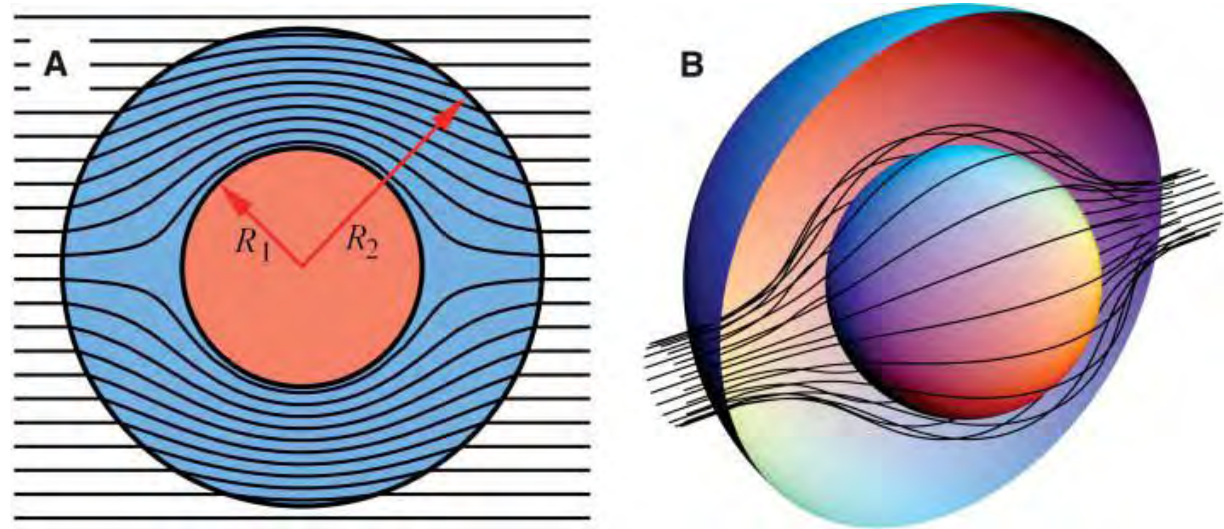
normal behavior





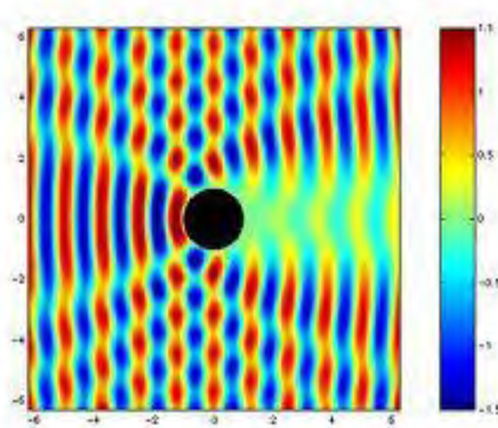
A cloak makes waves travel around an object

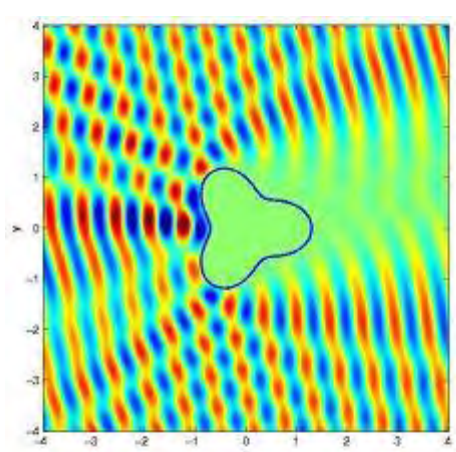




Pendry, Schurig and Smith
Science 2006

normal behavior



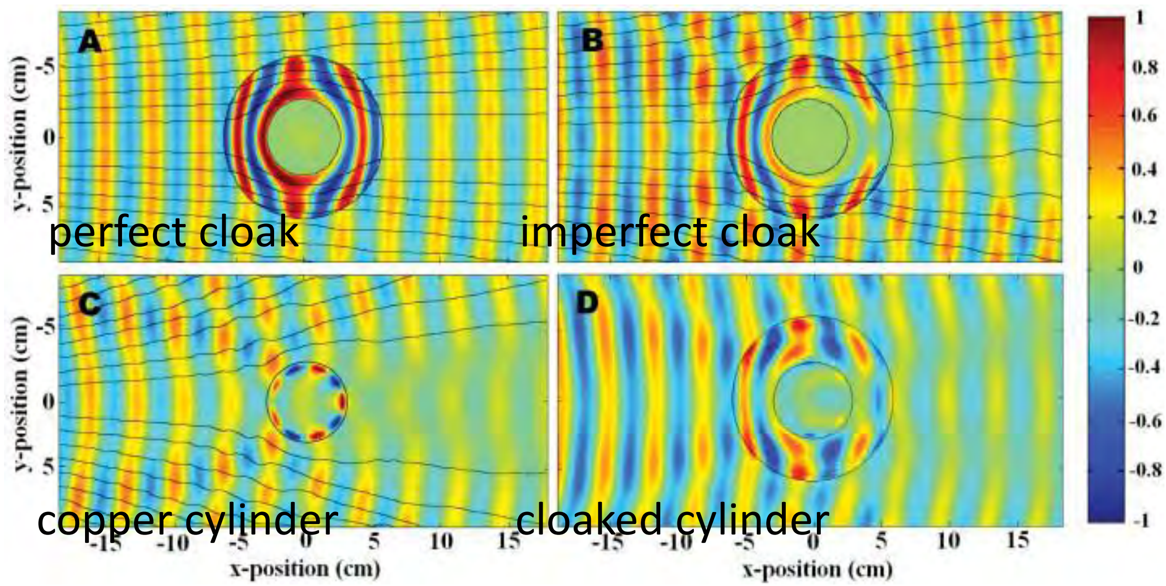


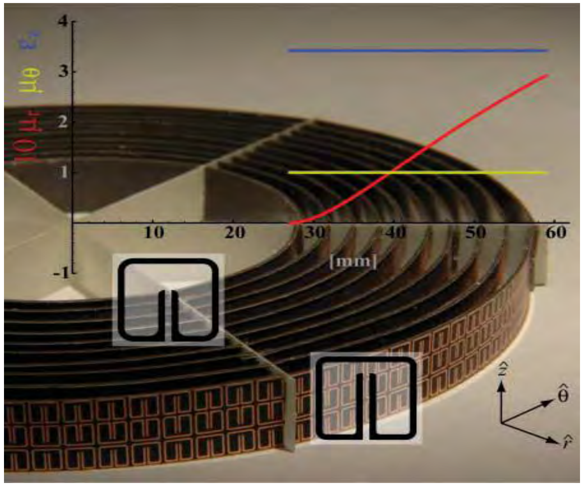
Metamaterial Electromagnetic Cloak at Microwave Frequencies

Schurig, Mock, Justice, Cummer, Pendry, Starr, Smith, *Science*, 2006

simulation

experiment





Why should engineers be interested in *metamaterials*?

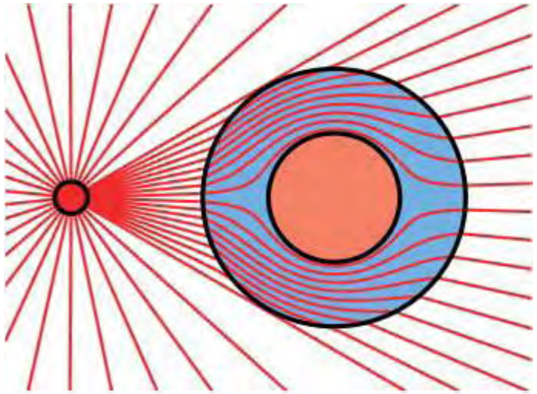
potential for radically new devices/technologies

- vibration insulation
- ultrasonic imaging beyond the diffraction limit
- control of SAWs: filtering, guiding, …
- stealth for underwater structures

why now?

combination of:

- cross-disciplinarity
- advanced fabrication techniques
- computational availability
- thinking "outside the box"



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waves in periodic systems

examples of phononic crystals

Acoustic cloaking – engineering the impossible

transformation acoustics

1D, 2D, cylindrical

inertial materials

Pentamode materials

cloaking elastic waves

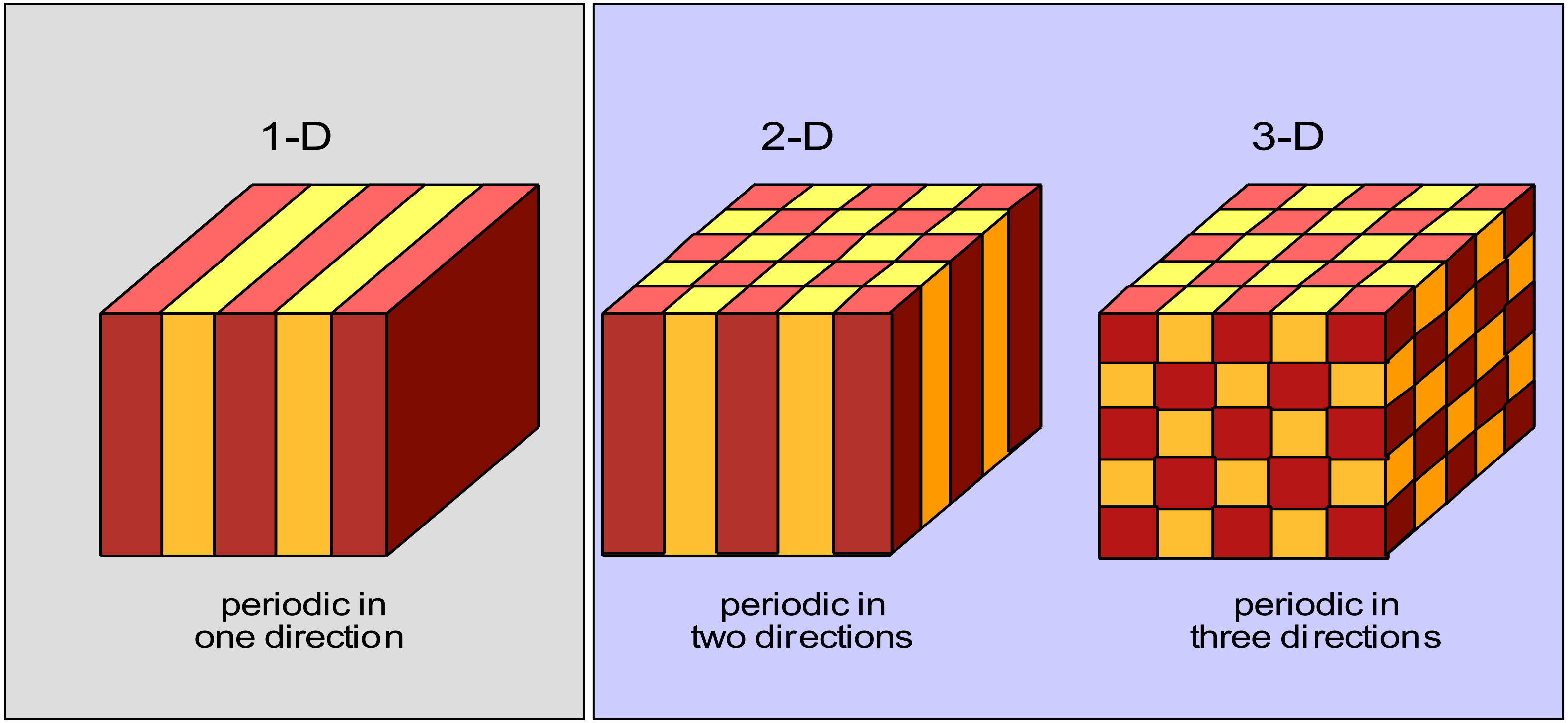
Metatheory for metamaterials?

phononic crystals

engineering the band gap

phononic crystals

periodic mechanical systems



phononic band gaps provide tools for controlling waves:
filtering, steering, trapping, focusing beyond the diffraction limit, etc.

acoustics

density ρ

bulk modulus K

pressure relation $\dot{p} = -K \,\mathrm{div}\, \mathbf{v}$

momentum balance $\rho\dot{\mathbf{v}} = -\nabla p$

assume constant $\rho \quad K$

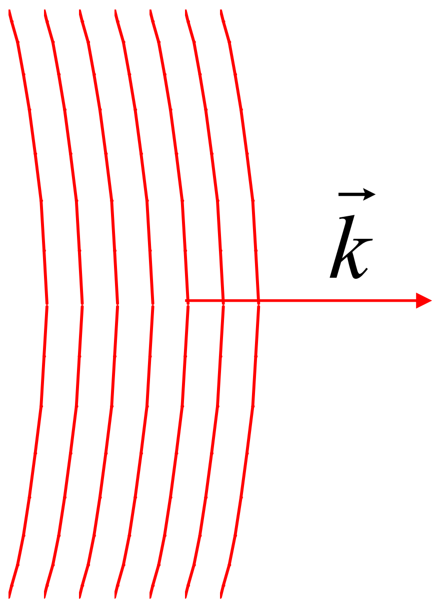
$$\Rightarrow \boxed{\nabla^2 p - \frac{1}{c^2}\ddot{p} = 0} \qquad c = \sqrt{\frac{K}{\rho}}$$

wave speed

water 1500 m/s
air 330 m/s

ratio of pressure to particle velocity in a plane wave $\frac{p}{v} = \rho c \equiv$ acoustic *impedance*

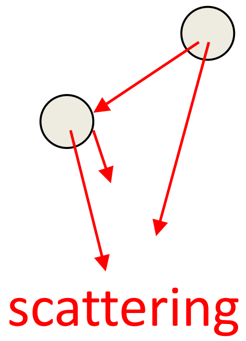
scattering of sound

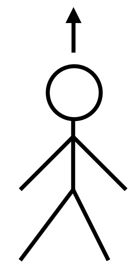


planewave

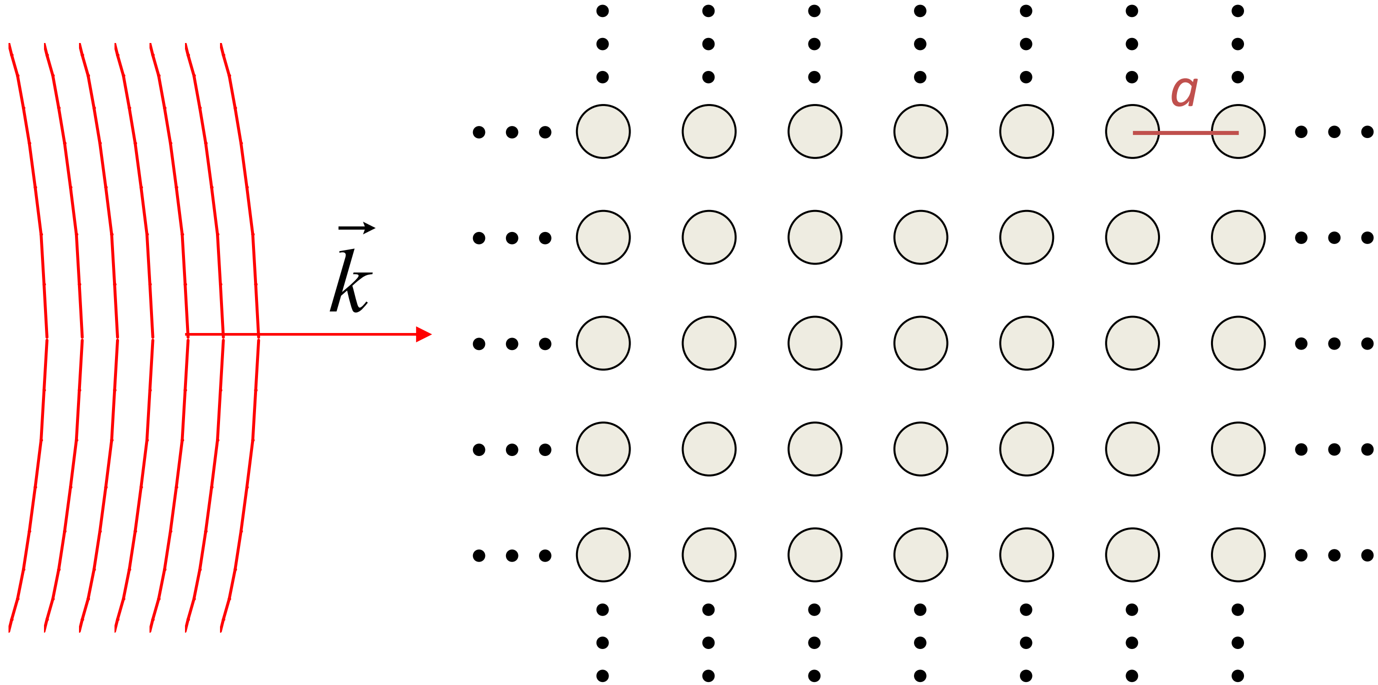
$$p \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$





periodic system



planewave

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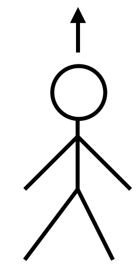
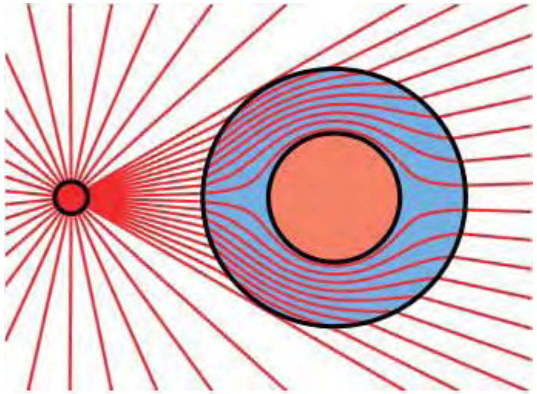


for most λ, sound propagates through crystal without scattering (scattering cancels coherently)

...but for some λ (~ $2a$), no sound propagates: **a phononic band gap**



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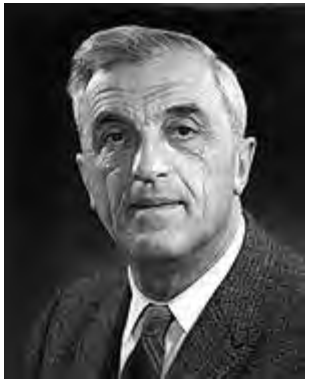
Metatheory for metamaterials?

waves in periodic systems

G. Floquet (1883) F. Bloch (1928)



F. Bloch

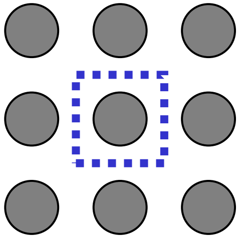
Bloch-Floquet theorem applies:

$$\mathbf{u}(\vec{\mathrm{x}}, t) = e^{i\left(\vec{k}\cdot\vec{x} - \omega t\right)} \mathbf{U}_{\vec{k}}(\vec{x})$$

plane wave periodic envelope

k is constant, *i.e.* no scattering of Bloch wave

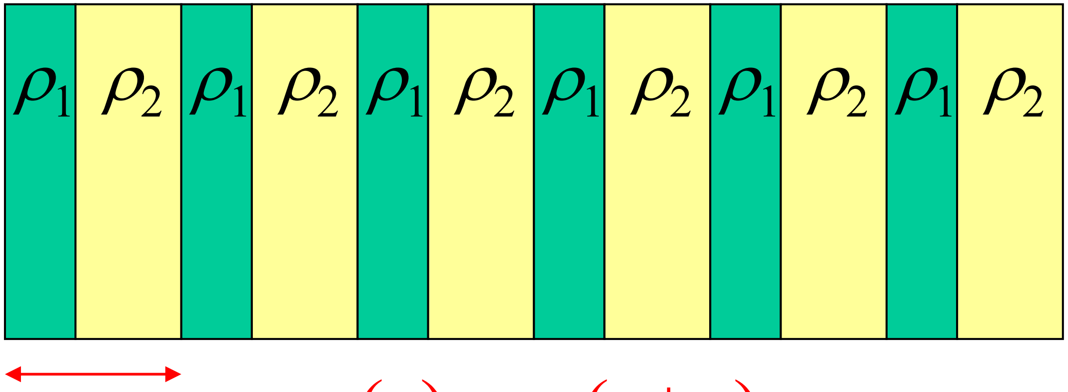
$\mathbf{U}_{\vec{k}}$ given by the finite unit cell,
so the frequencies are discrete $\omega_n(\mathbf{k})$



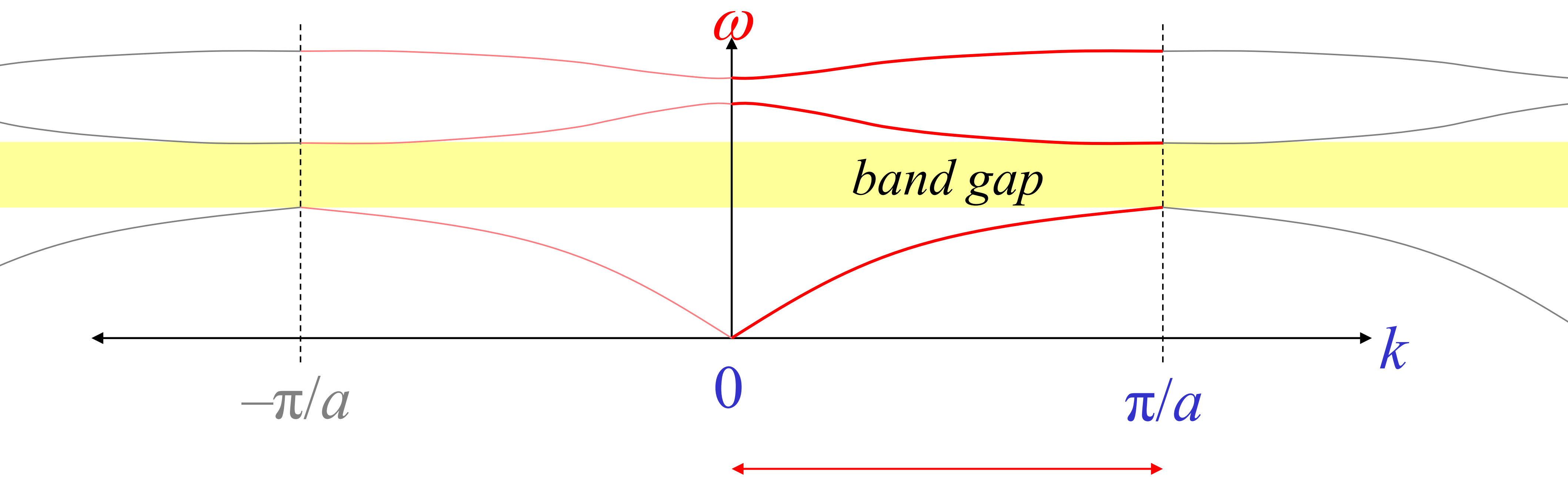
waves in periodic systems: 1D

k is periodic:

$k + 2\pi/a$ equivalent to k



$\rho(x) = \rho(x+a)$

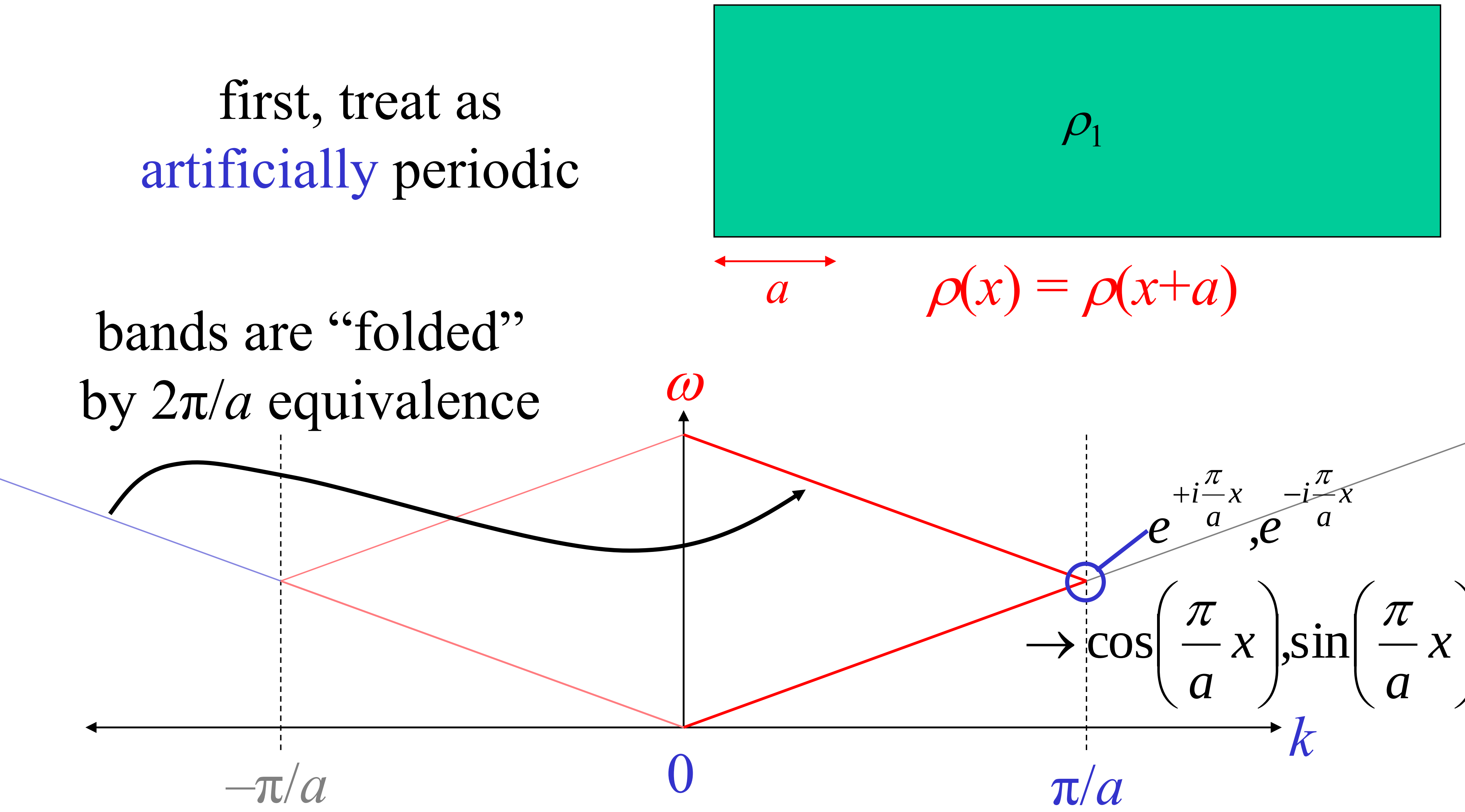


irreducible Brillouin zone

Any 1D periodic system has a gap

first, treat as
artificially periodic

bands are "folded"
by $2\pi/a$ equivalence

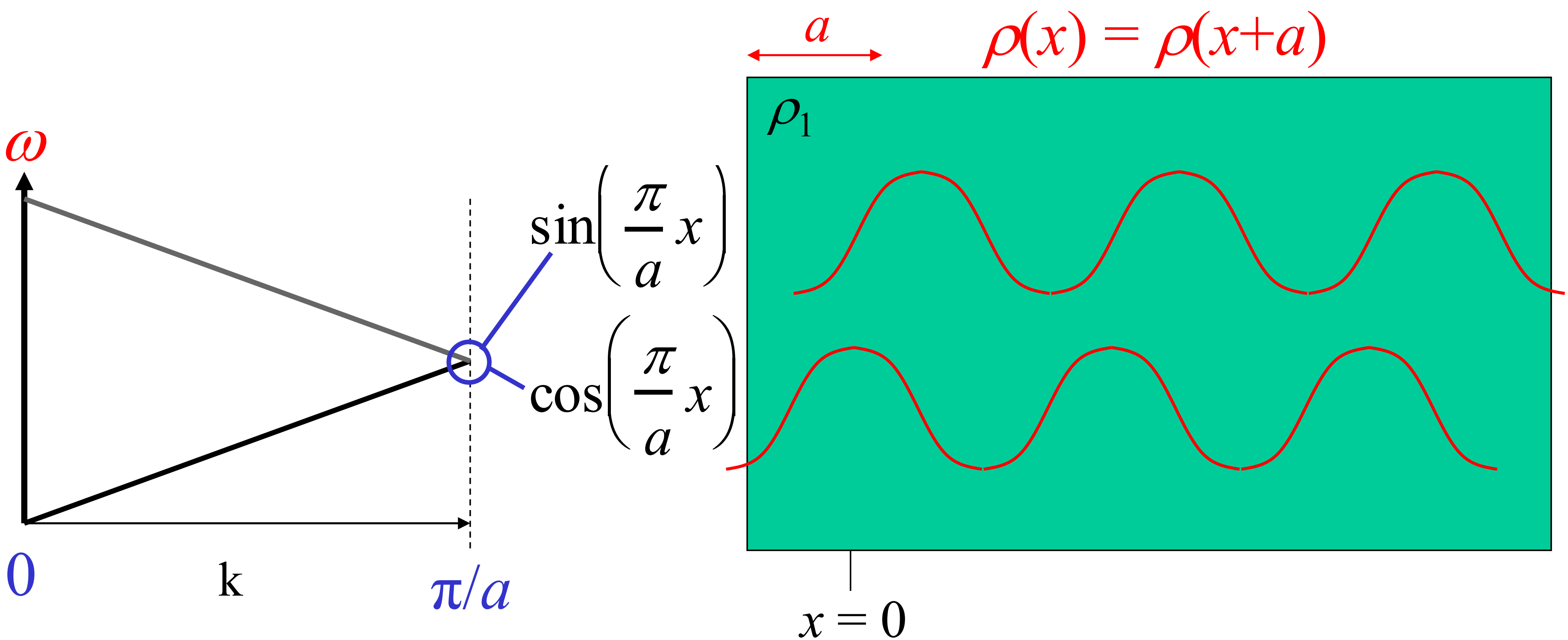


Any 1D periodic system has a gap

still treat it as "artificially" periodic

forward, backward propagating waves
or standing waves

$$u = u_0 \cos(kx - \omega t) + u_0 \cos(kx + \omega t) = 2u_0 \cos(kx)\,\cos(\omega t)$$

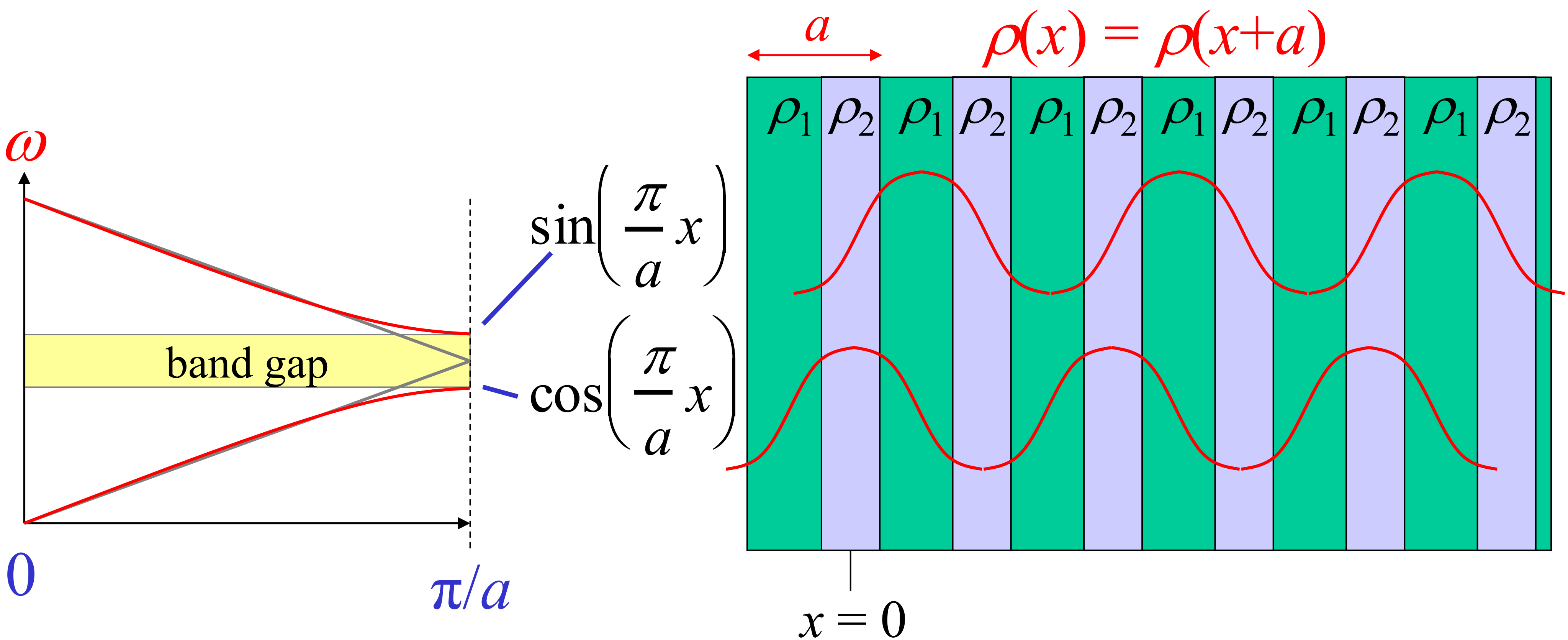


Any 1d periodic system has a gap

add a small "real" density periodicity

$$\rho_2 = \rho_1 + \Delta\rho$$

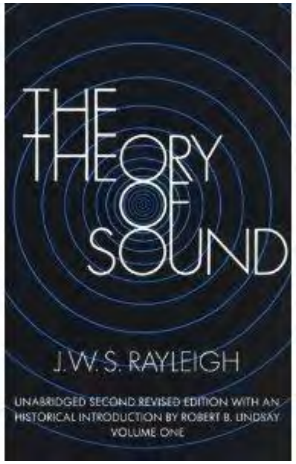
splitting of degeneracy:
wave mainly in slower zone (ρ_2) has lower frequency



Rayleigh predicted band gaps in 1887

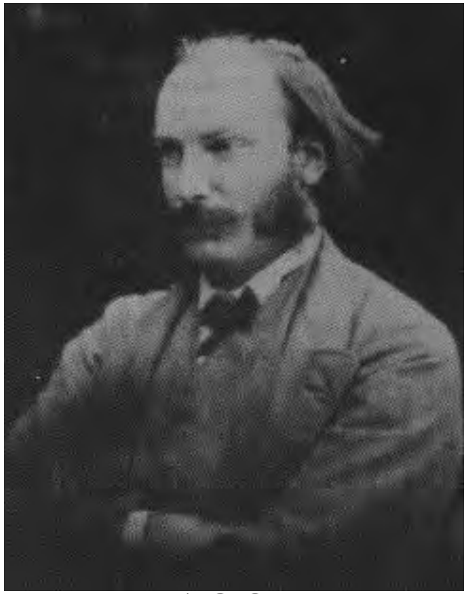
Rayleigh 1870

The meaning of this is that a wave travelling in either direction is ultimately totally reflected. For example, we may so choose the values of R and S that at the origin of x there is a wave (of given strength) in the positive direction only, and we may imagine that it here passes into a uniform medium, and so is propagated on indefinitely without change. But, in order to maintain this state of things, we have to suppose on the negative side the coexistence of positive and negative waves, which at sufficient distances from the origin are of nearly equal and ever-increasing amplitudes. In order therefore that a small wave may emerge at $x = 0$, we have to cause intense waves to be incident upon a face of the medium corresponding to a large negative x, of which nearly the whole are reflected.



ON THE MAINTENANCE OF VIBRATIONS BY FORCES OF DOUBLE FREQUENCY, AND ON THE PROPAGATION OF WAVES THROUGH A MEDIUM ENDOWED WITH A PERIODIC STRUCTURE.

[*Philosophical Magazine*, XXIV. pp. 145—159; 1887.]



1885

Bloch waves – 1D periodic system

time harmonic $e^{-i\omega t}$

$$-i\omega\rho v = -p'$$
$$-i\omega p = -Kv'$$

$K = zc$

$\rho = z/c$

$$\begin{pmatrix} v \\ p \end{pmatrix}' = i\omega \begin{pmatrix} 0 & K^{-1} \\ \rho & 0 \end{pmatrix} \begin{pmatrix} v \\ p \end{pmatrix} = i\frac{\omega}{c} \begin{pmatrix} 0 & z^{-1} \\ z & 0 \end{pmatrix} \begin{pmatrix} v \\ p \end{pmatrix}$$

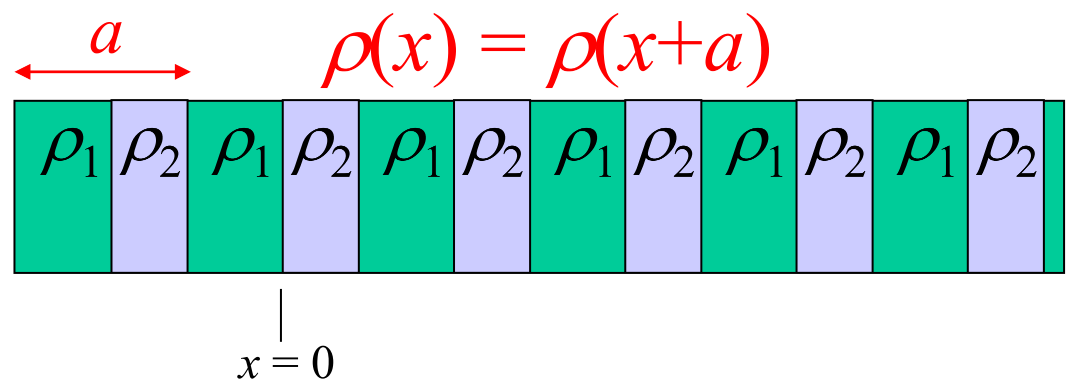
$\mathbf{A}^2 = \mathbf{I}$

$$\boldsymbol{\eta}' = ik\mathbf{A}\boldsymbol{\eta}$$
$$\boldsymbol{\eta}(x) = e^{ikx\mathbf{A}}\boldsymbol{\eta}(0)$$

propagator $e^{ikx\mathbf{A}} = \mathbf{I}\cos kx + i\mathbf{A}\sin kx$

$\det e^{ikx\mathbf{A}} = 1$

$\rho(x) = \rho(x+a)$



two phase medium

Bloch condition $\boldsymbol{\eta}(a) = e^{ika}\boldsymbol{\eta}(0)$

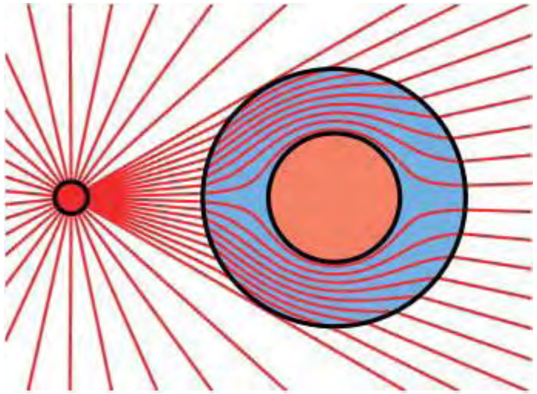
$k_1 = \frac{\omega}{c_1}$ $k_2 = \frac{\omega}{c_2}$

$$\left(e^{ik_1a_1\mathbf{A}_1}e^{ik_2a_2\mathbf{A}_2} - e^{ika}\mathbf{I}\right)\boldsymbol{\eta}(0) = 0$$

$$e^{ika} + e^{-ika} = \mathrm{tr}\left(e^{ik_1a_1\mathbf{A}_1}e^{ik_2a_2\mathbf{A}_2}\right)$$

dispersion relation

$$\cos ka = \cos k_1a_1 \cos k_2a_2 - \sin k_1a_1 \sin k_2a_2 \frac{1}{2}\left(\frac{z_1}{z_2} + \frac{z_2}{z_1}\right))$$



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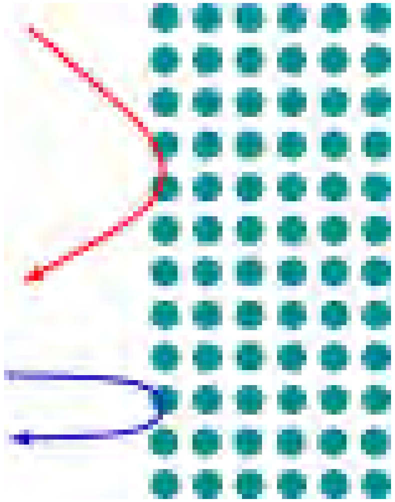
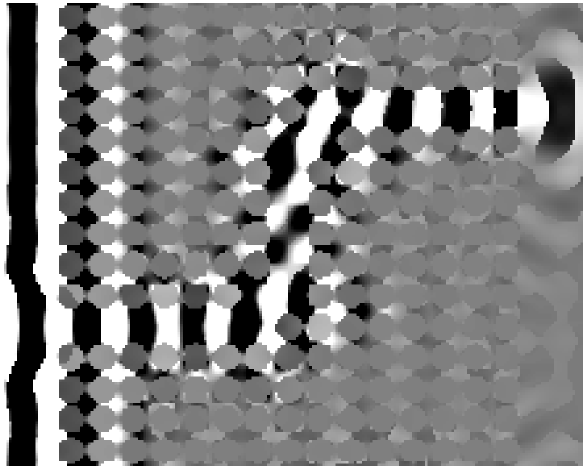
example applications

filtering and controlling waves





Khelif et al (APL 2004)

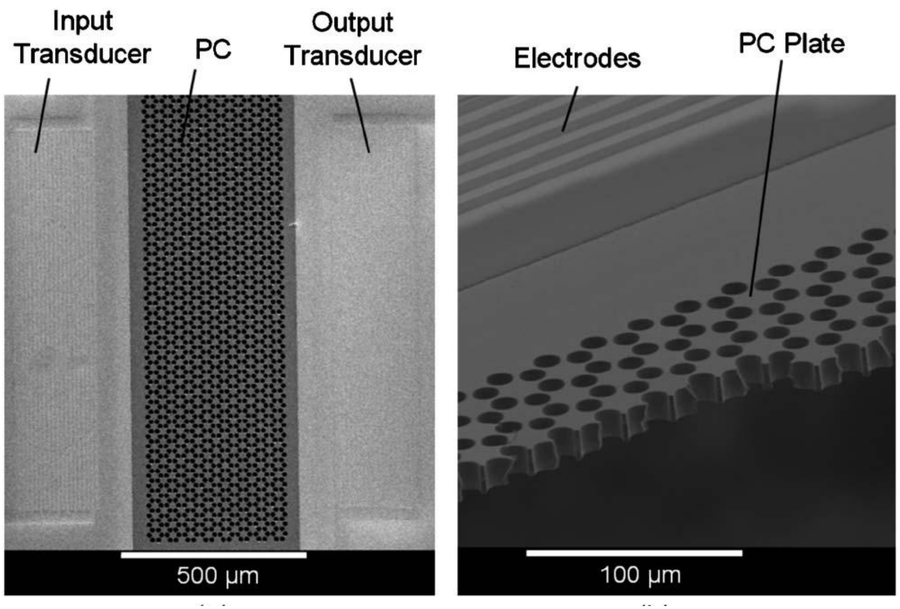
perfect mirror (in band gap)

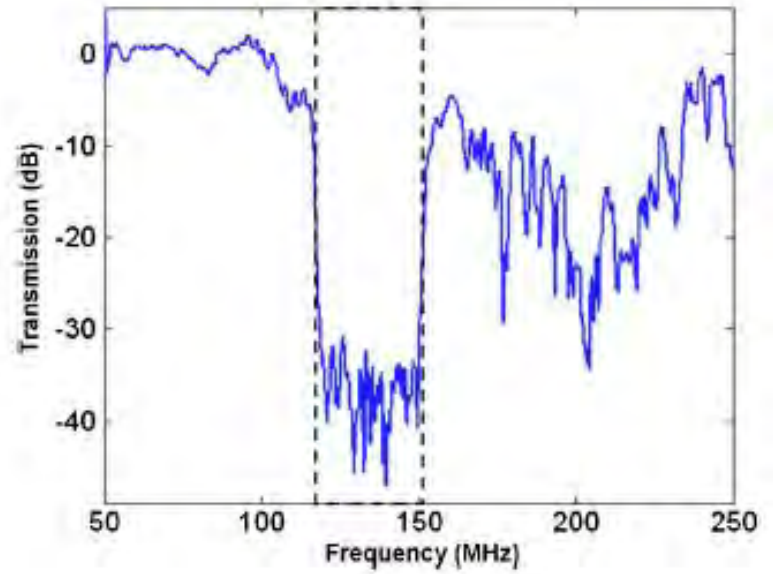
- frequency filter

wave guide

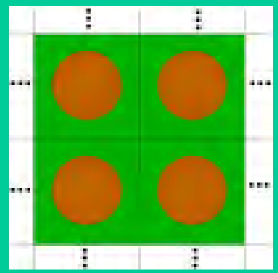
- beam splitting, multiplexing

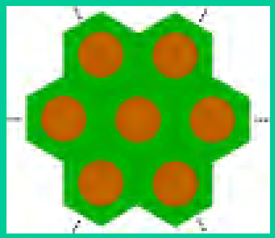
phononic crystals for surface acoustic waves



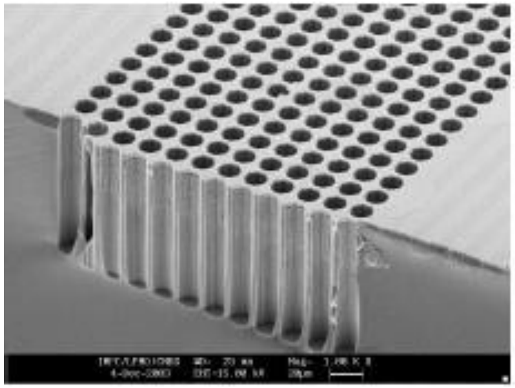


Mohammadi et al. (APL 2008)



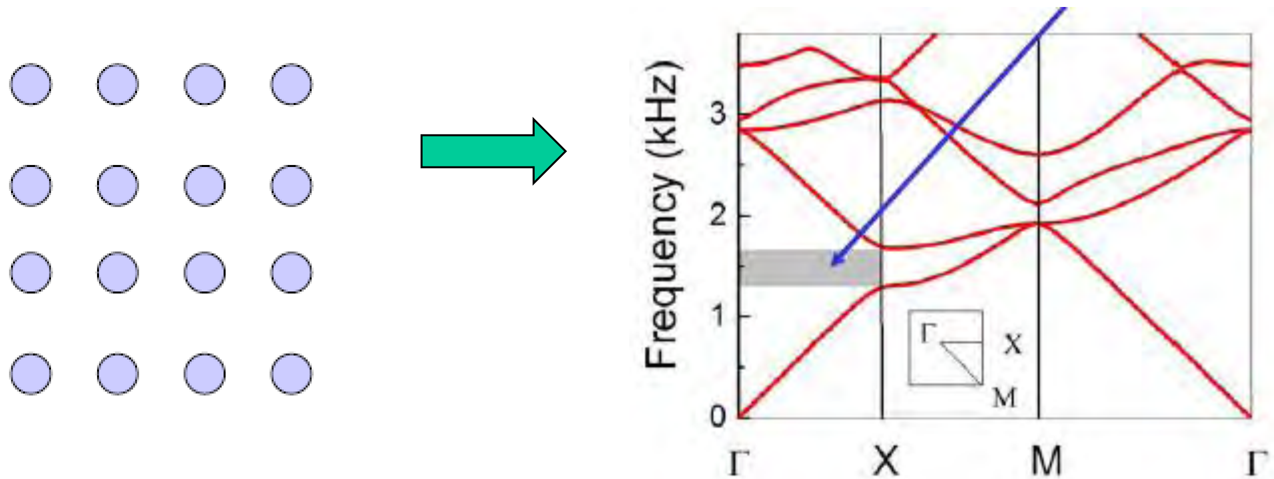


cubic arrays give larger gaps than triangular but are not as isotropic

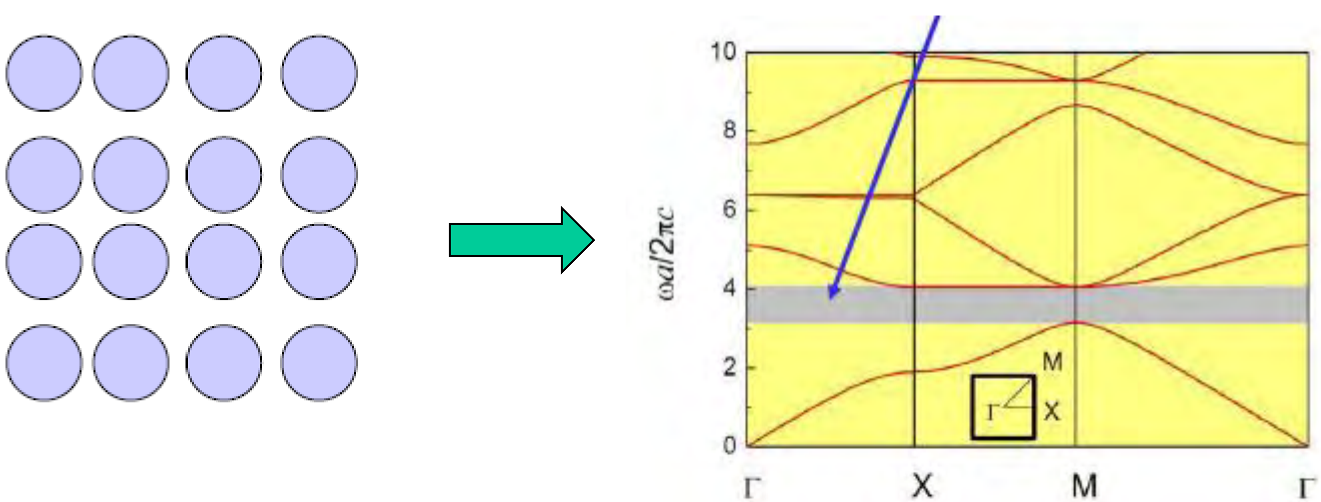


stronger scattering leads to greater band gaps

partial band gap for waves in some directions only

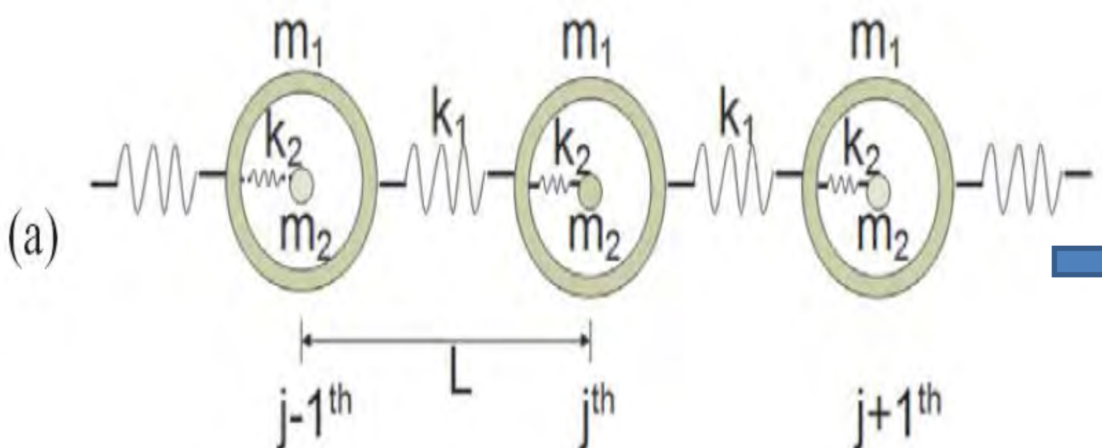


Complete band gap - all directions

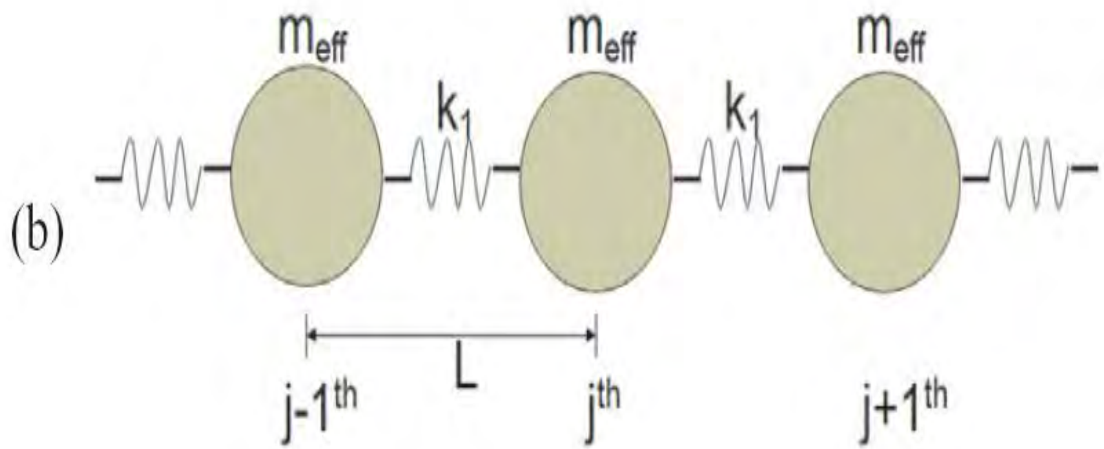


resonant oscillator metamaterials

mass-in mass system



equivalent system



Compare equilibrium eqns:

$$\frac{m_{\mathrm{eff}}}{m_{\mathrm{st}}} = 1 + \left(\frac{\theta}{1+\theta}\right) \frac{(\omega/\omega_0)^2}{1-(\omega/\omega_0)^2}.$$

$$\theta = m_2/m_1 \qquad \omega_0^2 = k_2/m_2$$

$$m_{\mathrm{st}} = m_1 + m_2$$

Negative effective mass for

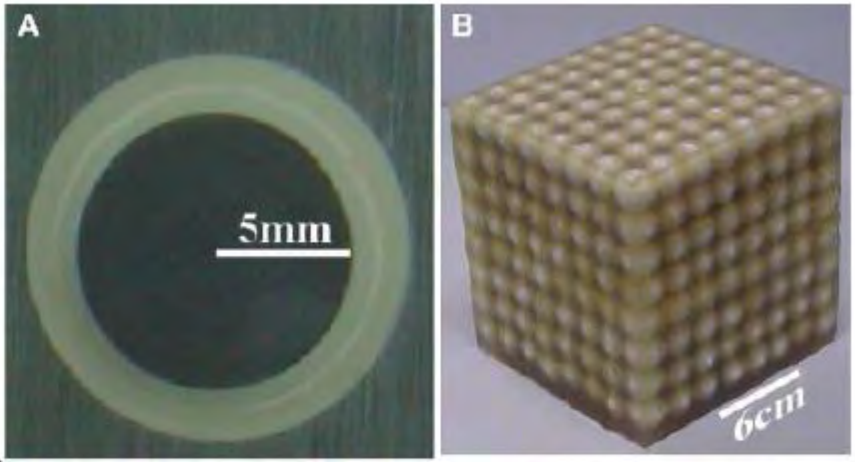
$$1 < (\omega/\omega_0)^2 < 1 + \theta.$$

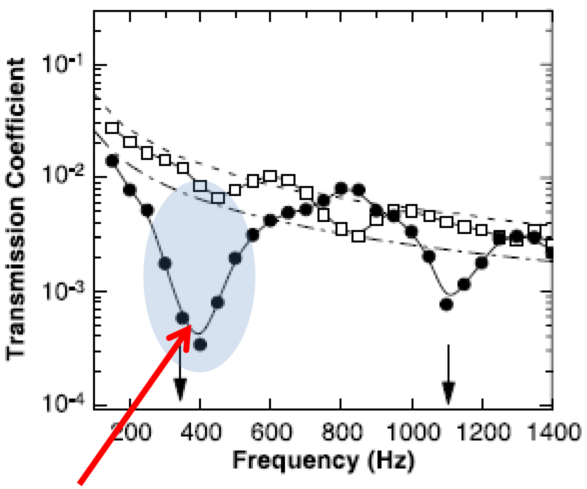
Control bandgap frequencies

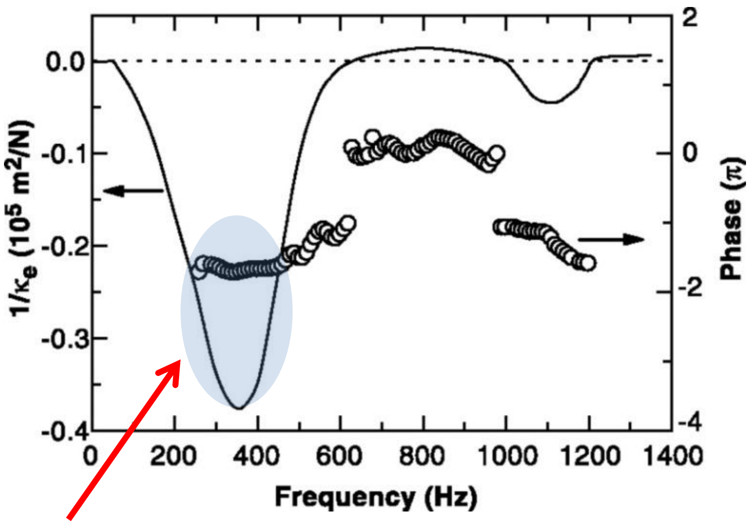
H. H. Huang, C. T. Sun, New J. Phys. 11 2009

low frequency band gap: heavy & soft

Lead coated spheres + layer of silicone rubber in cubic array







very low frequency band gap and **negative elastic constant** due to dipole resonance

sub-wavelength band gap **λ >> a**

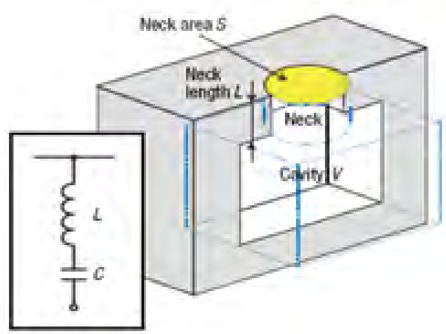
Z. Liu, X. Zhang, Y. Mao, Y.Y. Zhu, Z. Yang, C.T. Chan, P. Sheng (Science 2000)

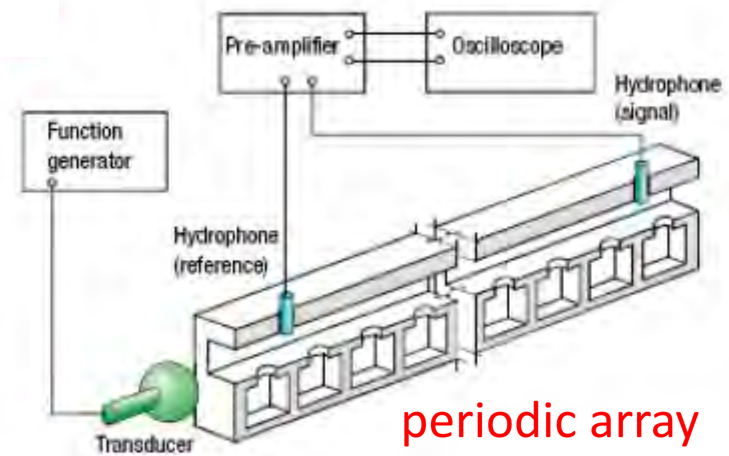
negative effective modulus

unit cell = Helmholtz resonator

$$\omega_0 \approx c\sqrt{S/L'V}$$

V = cavity vol,
L' = neck length,
S= neck X-section

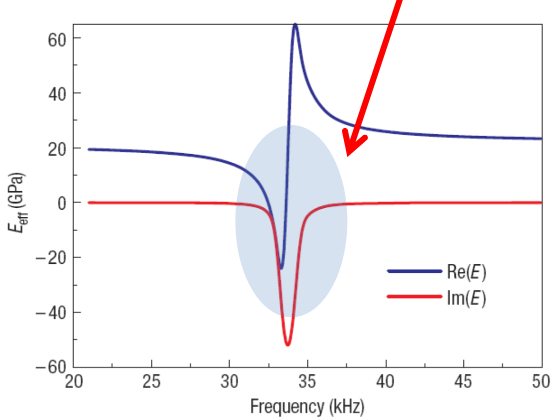


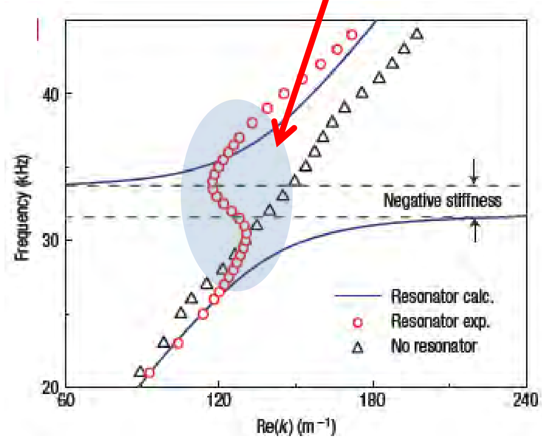


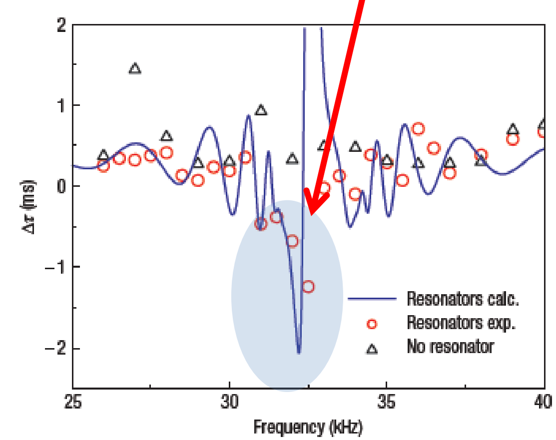
periodic array

effective modulus: $$E_{\text{eff}}^{-1}(\omega) = E_0^{-1}\left[1 - \frac{F\omega_0^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}\right]$$

negative **effective modulus** gives negative **group velocity** leading to **group delay**







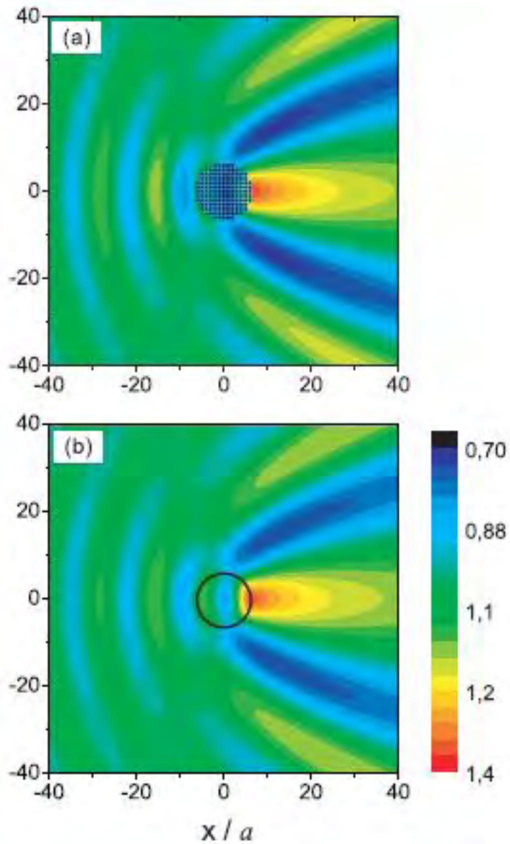
N. Fang et al. (2006 *Nature Mater.* **5** 452-6)

2D sonic crystals with tailored properties

lensing effect

plane wave scattering from finite array of regularly spaced identical cylinders

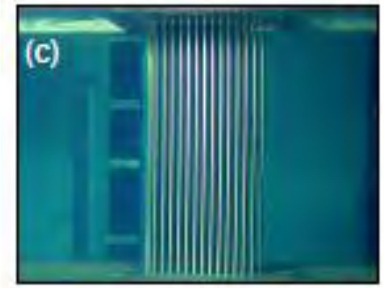
equivalent material with radially varying sound speed

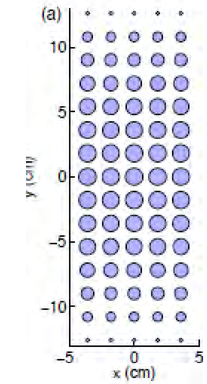


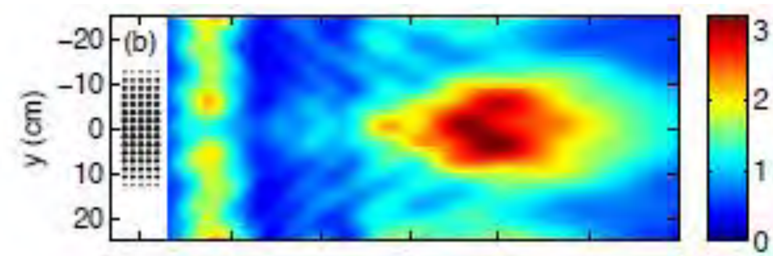
- uses multiple scattering theory (MST)

Sánchez-Dehesa, Torrent, L-W Cai (NJP 2009)

gradient index lens

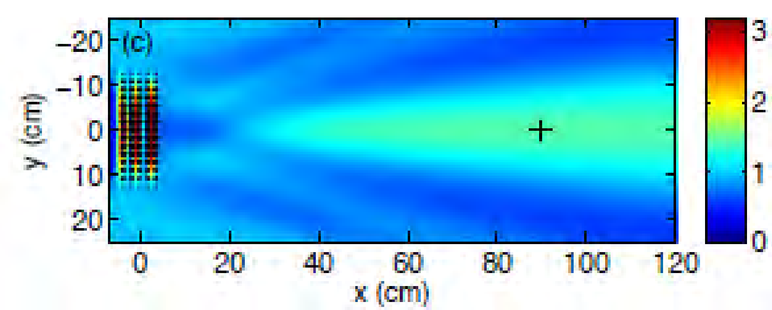






expt

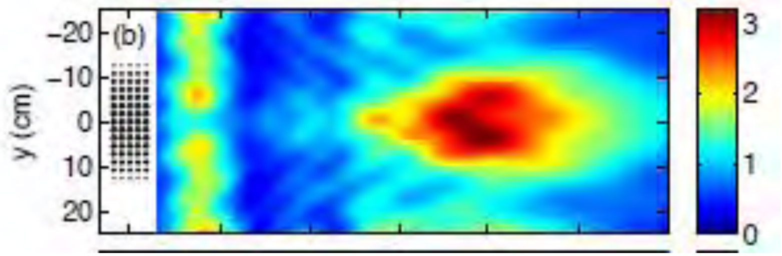
plane wave focusing



MST

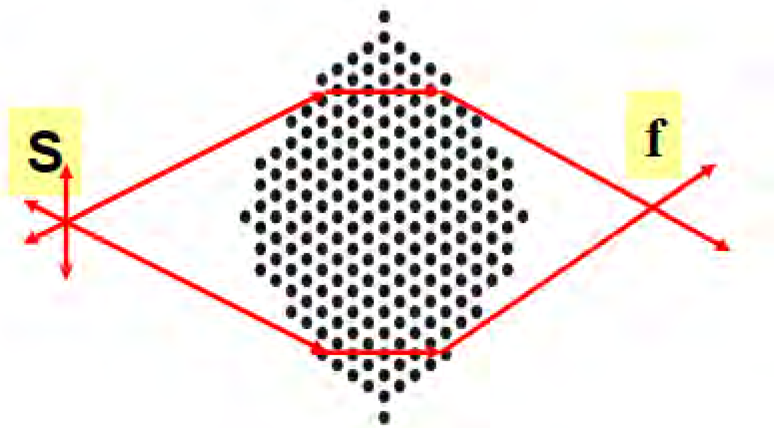
Martin et al. (APL 2010)

mechanism of sonic crystal focusing



dense solid cylinders in lighter fluid leads to **effective medium** with

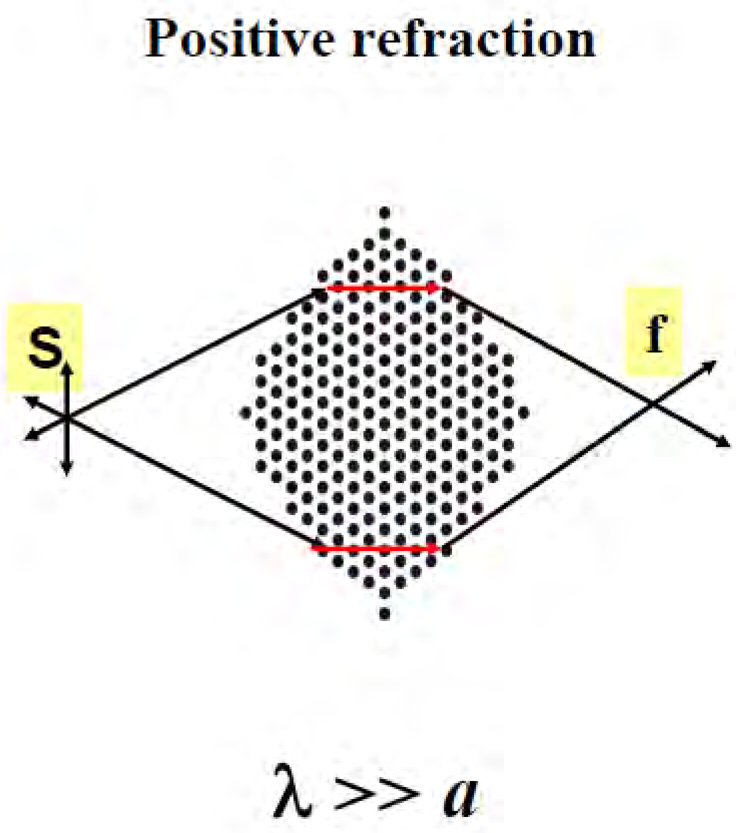
$$v_{eff} < v_{acoustic}$$

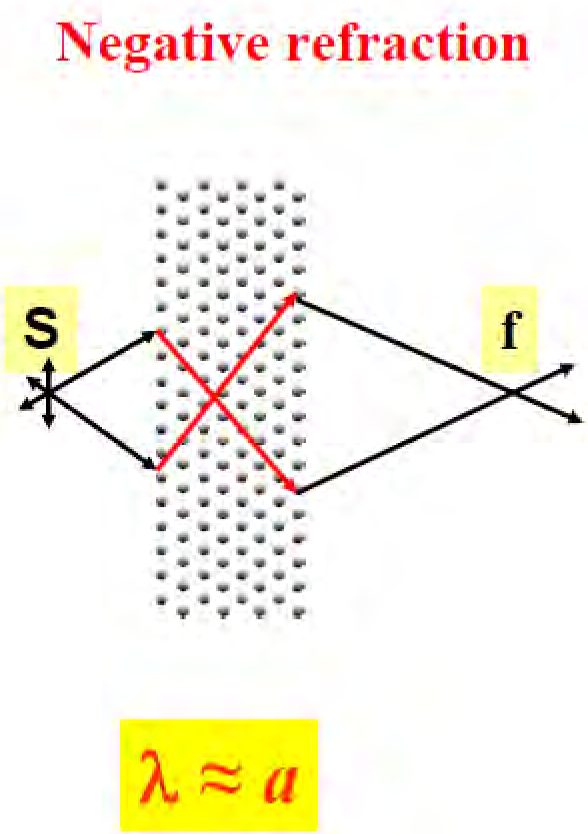


at the same time the effective impedance is not much different

→ "normal" lens effect

phononic crystals can lead to negative refraction

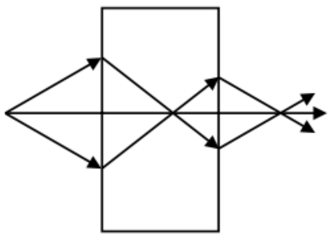




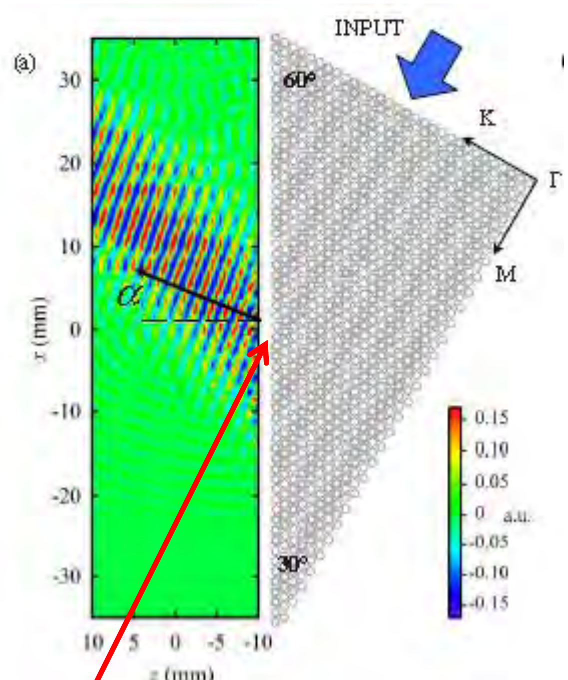
potential for resolution beyond the diffraction limit

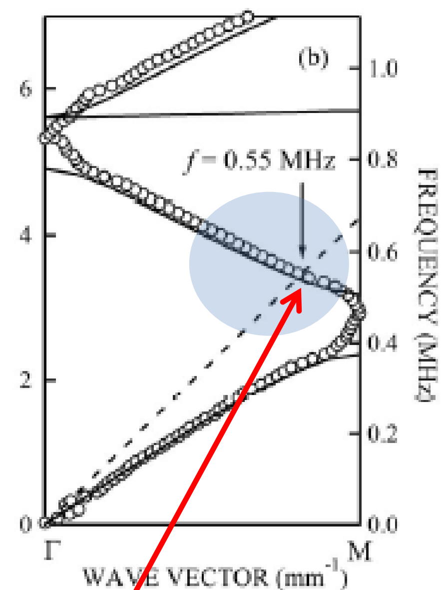
“perfect lens”

negative index for sound in water: fluid matrix NIM

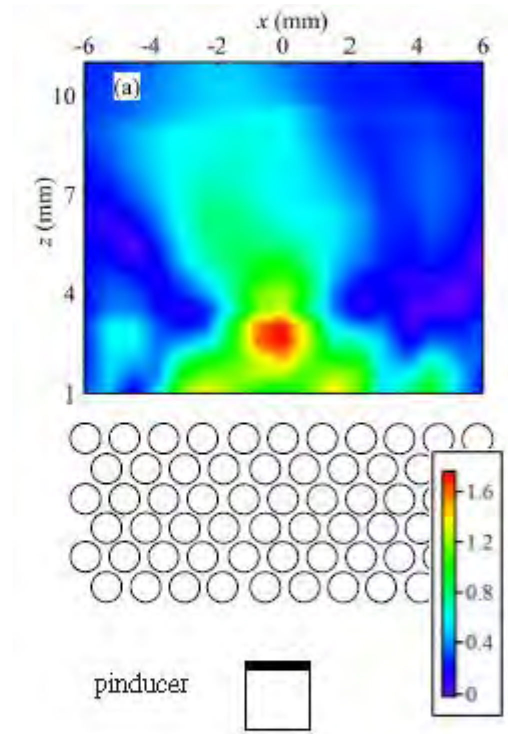


Sukhovich, Jing, Page (PRB 2008)



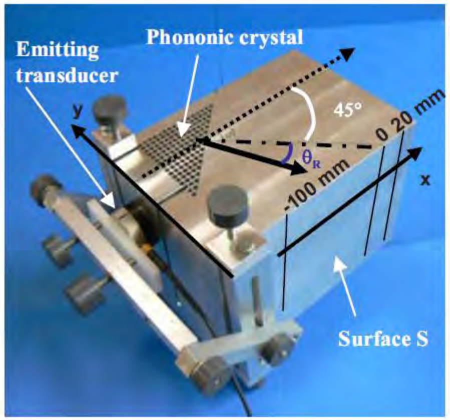


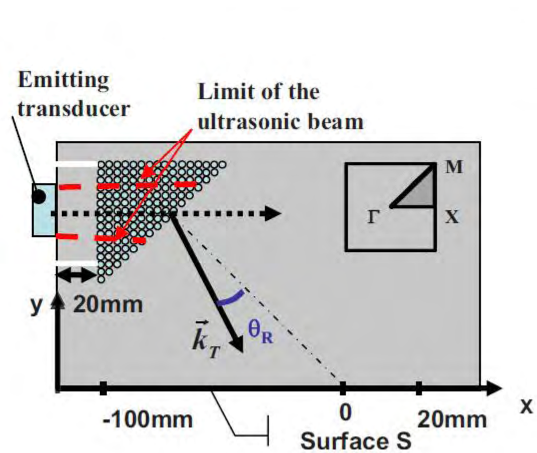
phase matching to **negative group velocity**



ultrasonic negative index
lens at 0.55 MHz

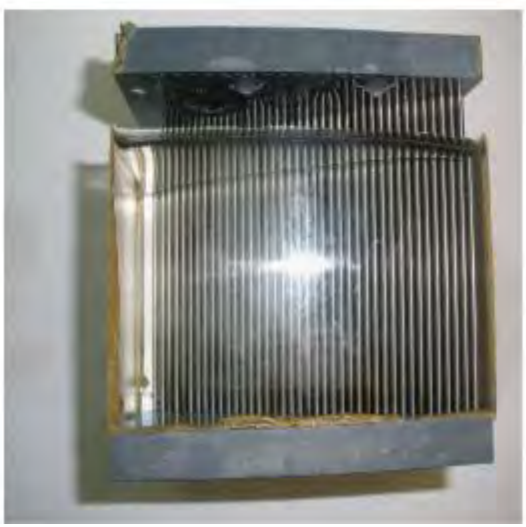
solid matrix negative index materials

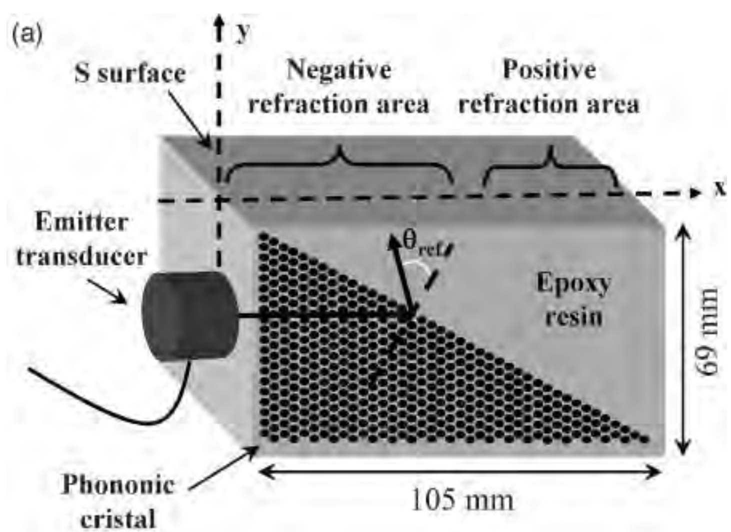




Metal matrix

Morvan et al. APL 2010





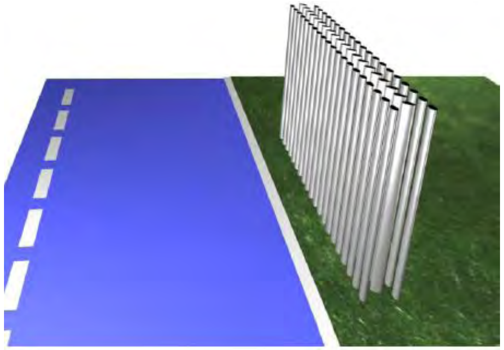
Epoxy matrix

Croenne et al. PRB 2011

Tuning properties to efficiently couple with sound in water remains a challenge

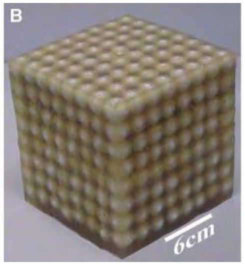
phononic crystals in engineering applications

acoustic insulation e.g. sound barriers using inexpensive materials

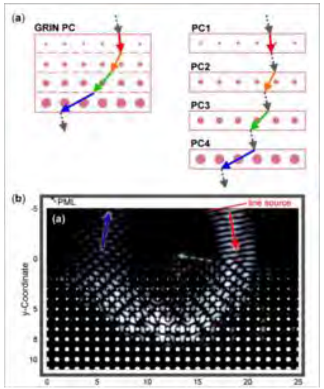


(Sanchez-Dehesa et al. ’10)

low frequency vibration isolation, using internal resonators

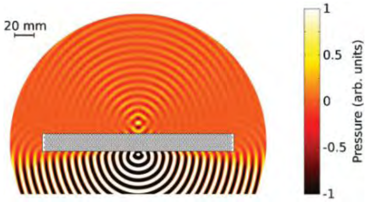


waveguides, SAW filters great potential, possible to fabricate, but radio-frequency devices still limited by energy loss



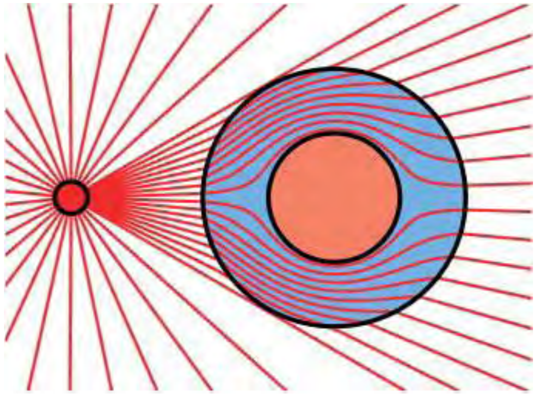
(Lin et al. JAP ’09)

negative index materials, lens work well in theory →
could provide super focusing, e.g. biomedical imaging
- not yet practical: impedance mismatch, material limits



(Croenne et al. ’11)

minimalist sculpture by E. Sempere (1923-1985) in a Madrid park, was demonstrated to be a phononic crystal in 1995



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Acoustic cloaking – engineering the impossible

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1D, 2D, cylindrical

inertial materials

Pentamode materials

cloaking elastic waves

Metatheory for metamaterials?

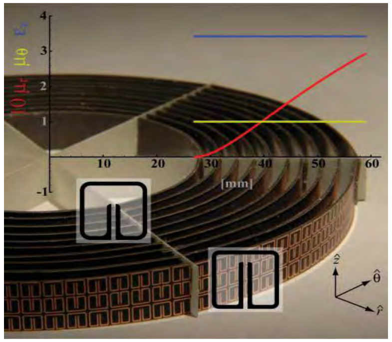
cloaking

engineering the impossible

background

Electromagnetic

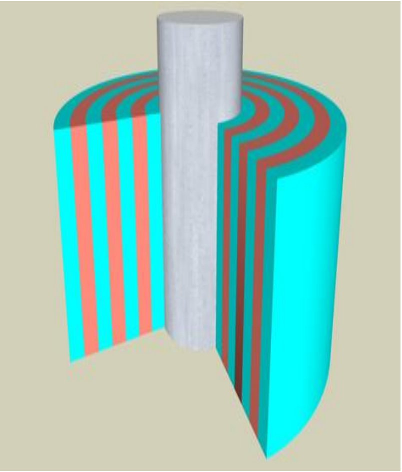
equations are invariant under *transformation* of coordinates - old idea



- *singular transformations* yield cloaking

- 2D cloaking demonstrated at microwave frequency
  Schurig et al ('06)

Acoustic



-*acoustic* cloaking 2D and 3D models using singular transformation
Chen & Chan ('07), Cummer et al ('07, '08)

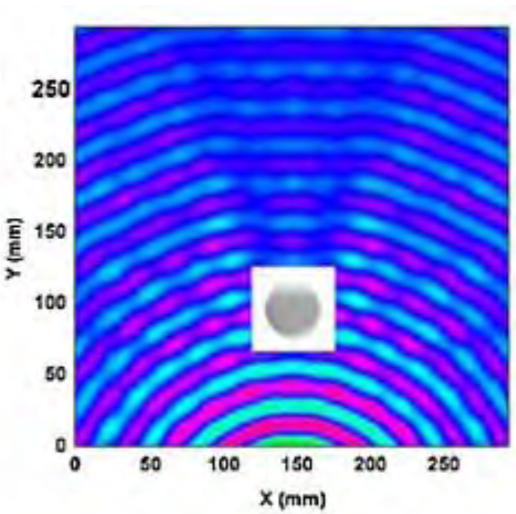
-these models are restricted to *anisotropic inertia*
-general theory allows *pentamode material* *(*Norris ('08, '09)

practical demonstration of:
broadband ultrasonic cloak *(*Zhang et al., PRL 2010)
acoustic carpet cloak *(*Popa et al., PRL 2011)

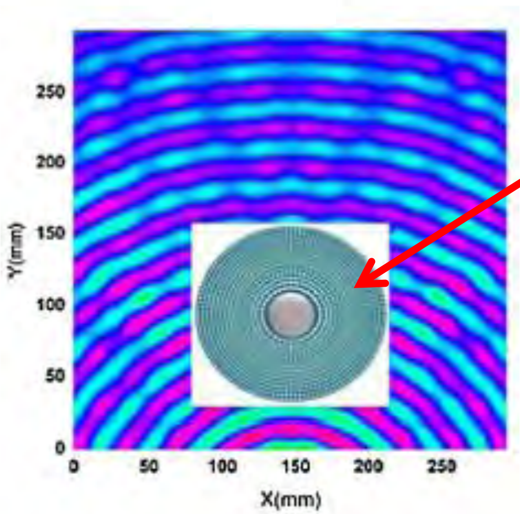
broadband cloaking demonstration

S. Zhang, C. Xia and N. Fang (PRL 2011)

60 kHz



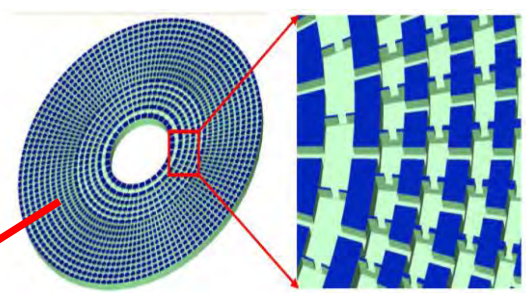
no cloak



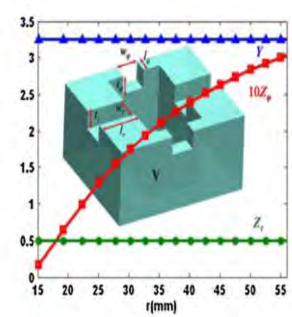
with cloak

2D transmission line approach

uses sub-wavelength acoustic Helmholtz resonator



(a)



(b)

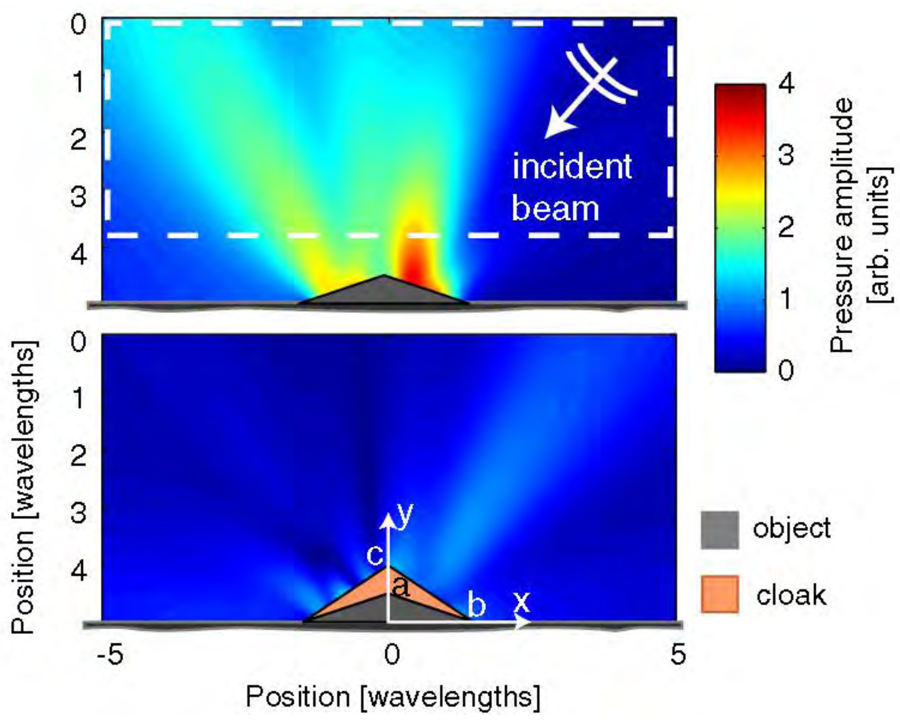
Layer	l_r(mm)	l_φ(mm)	V(mm^3)
1	2.05	0.10	3.00
3	1.37	0.22	2.29
5	1.24	0.41	2.06
7	1.24	0.30	2.06
9	1.24	0.41	2.06
11	1.24	0.52	2.06
13	1.24	0.63	2.06
15	1.24	0.74	2.06

(c)

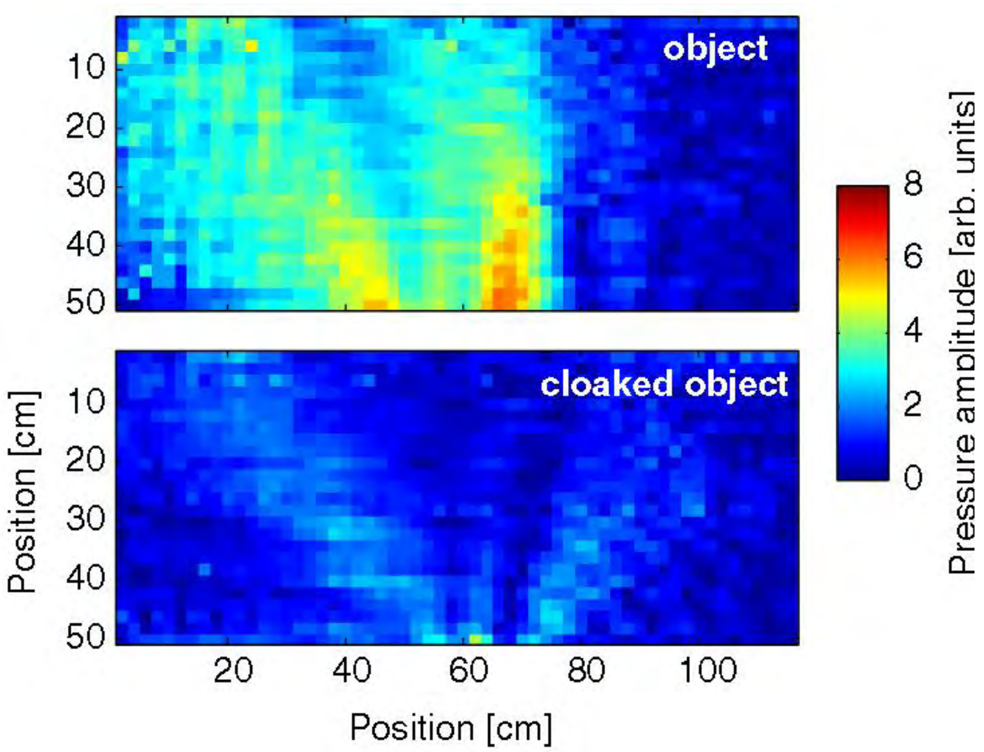
Acoustic **carpet cloak** in air

Popa, Zigoneanu, Cummer (PRL June 2011)

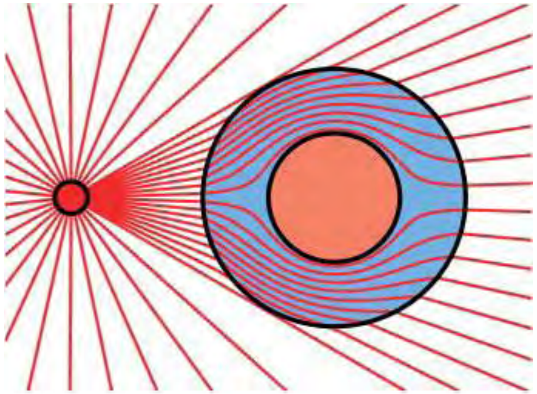
Idea of “carpet cloak”: cover the body so it looks like there is nothing there



theory/simulation



experiment at sonic frequency



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transformation acoustics

1D, 2D, cylindrical

inertial materials

Pentamode materials

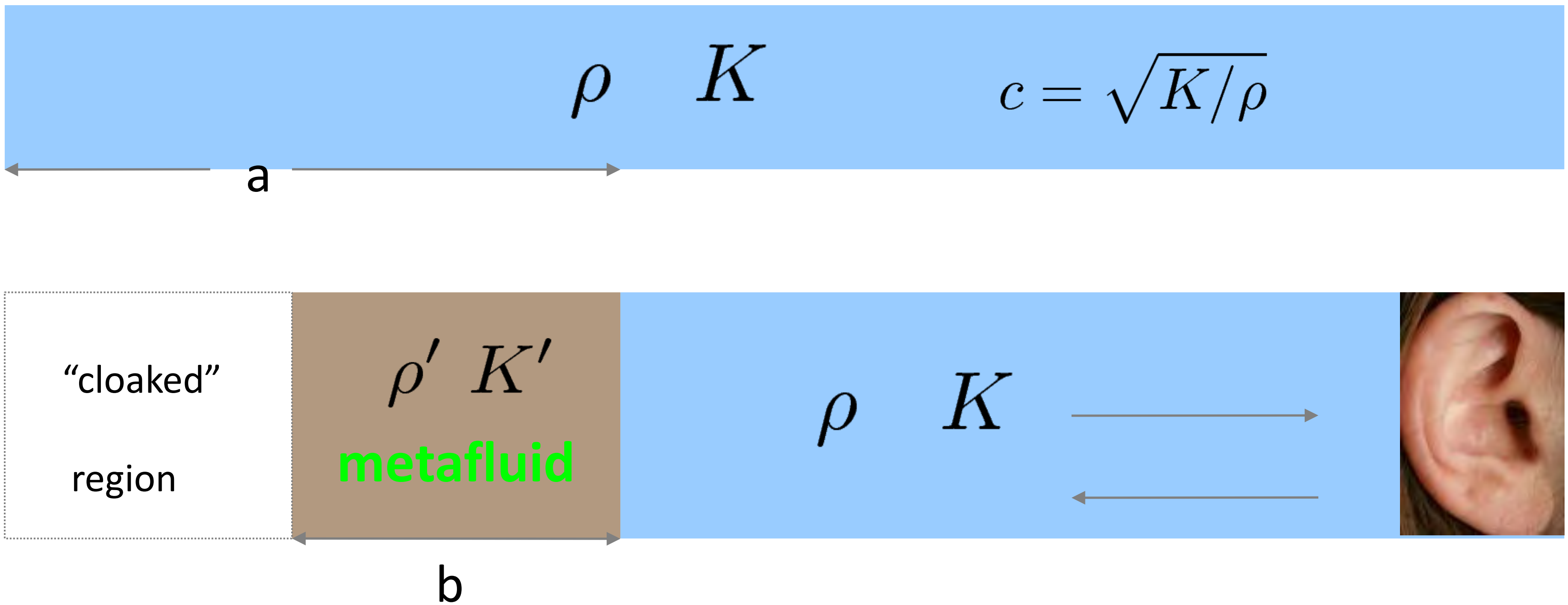
cloaking elastic waves

Metatheory for metamaterials?

cloaking

transformation acoustics

1D acoustic cloak mirage/illusion



same impedance/
no reflection

$$\rho' c' = \rho c$$

same travel time

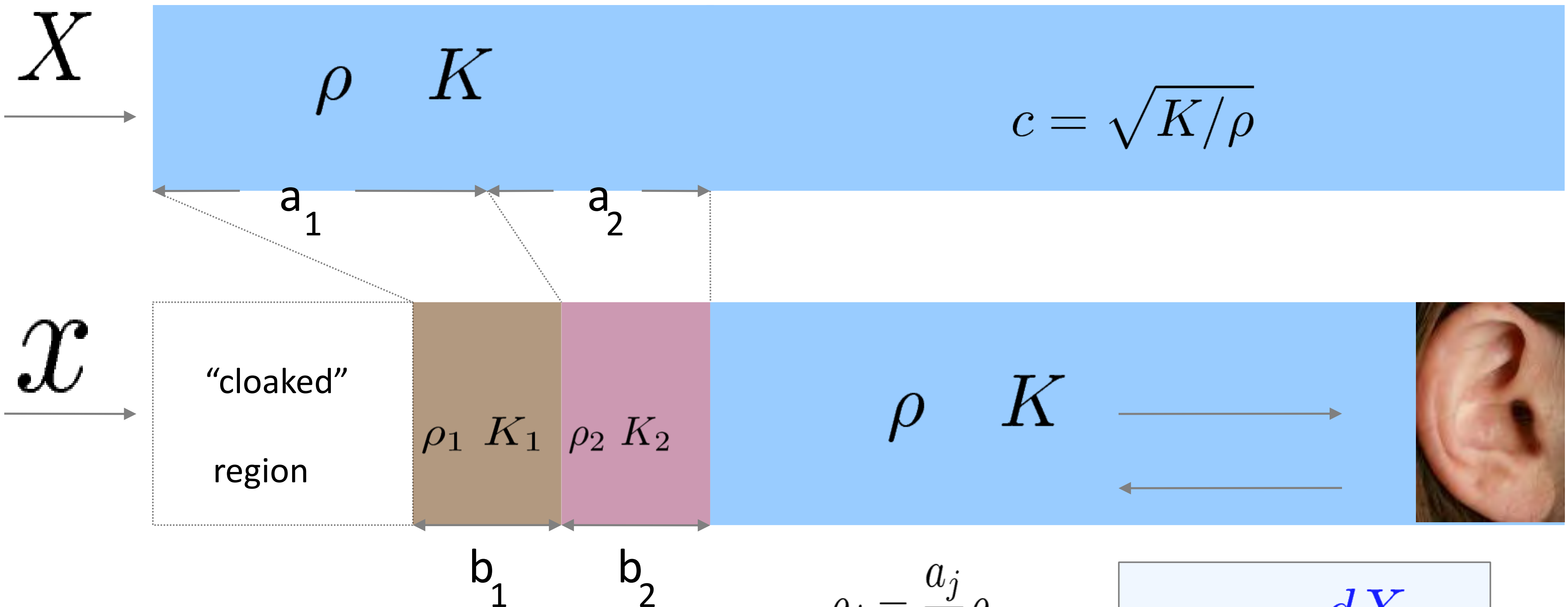
$$\frac{b}{c'} = \frac{a}{c}$$

$$\Rightarrow$$

$$\rho' = \frac{a}{b}\rho$$

$$K' = \frac{b}{a}K$$

idea of transformation: **geometrical mapping defines the material properties**



same impedance/ no reflection $\quad \rho_j c_j = \rho c$

same travel times $\quad \dfrac{b_j}{c_j} = \dfrac{a_j}{c}$

$$\Rightarrow \quad \rho_j = \frac{a_j}{b_j}\rho \qquad K_j = \frac{b_j}{a_j}K$$

$$\rho' = \rho \frac{dX}{dx}$$

$$K' = K \frac{dx}{dX}$$

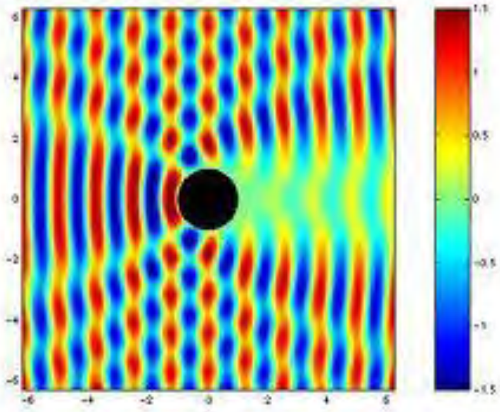
transformation $\Longrightarrow$ **material properties**

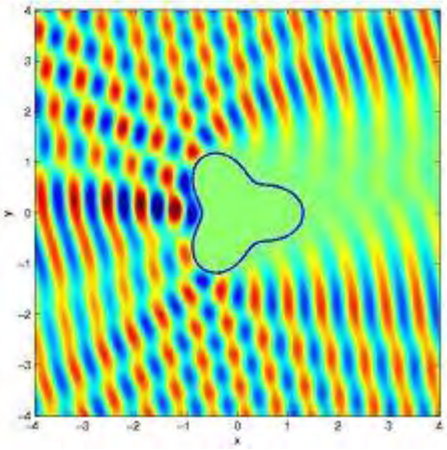
total mass is conserved

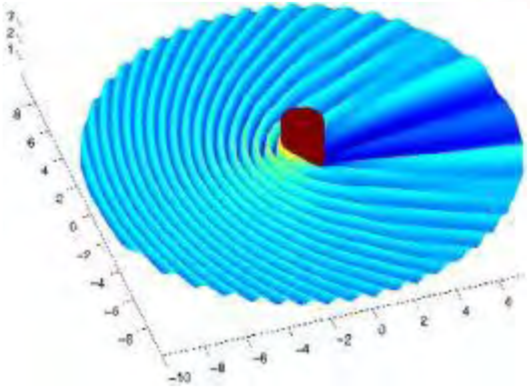
$$\int \rho' dx = \int \rho dX$$

scattering of waves

= reflection in all directions

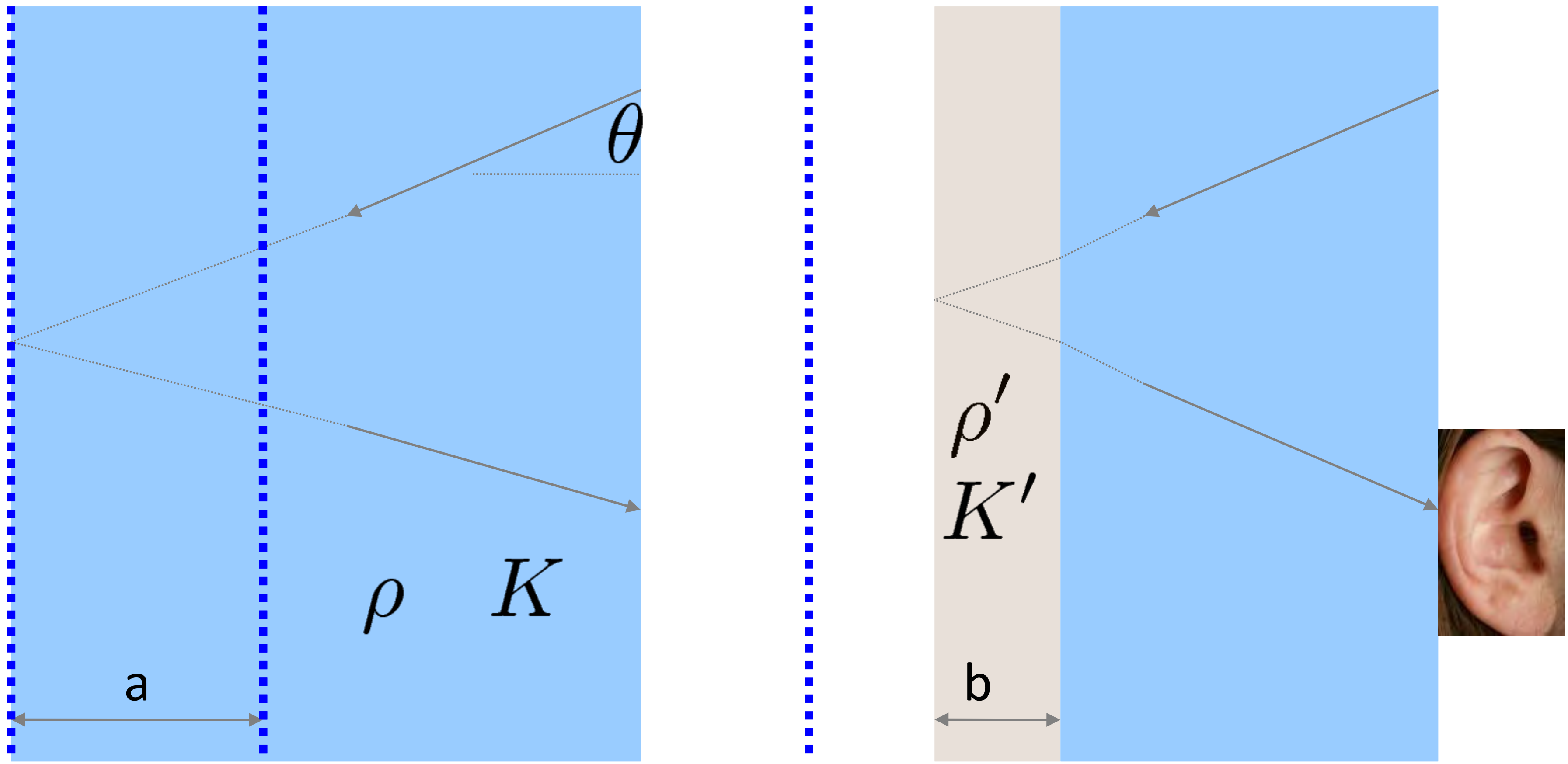






much harder !

2D acoustic mirage



impedance $$\frac{\rho' c'}{\cos\theta'} = \frac{\rho c}{\cos\theta}$$

travel time $$\frac{b}{c'\cos\theta'} = \frac{a}{c\cos\theta}$$

Snell $$\frac{1}{c'}\sin\theta' = \frac{1}{c}\sin\theta$$

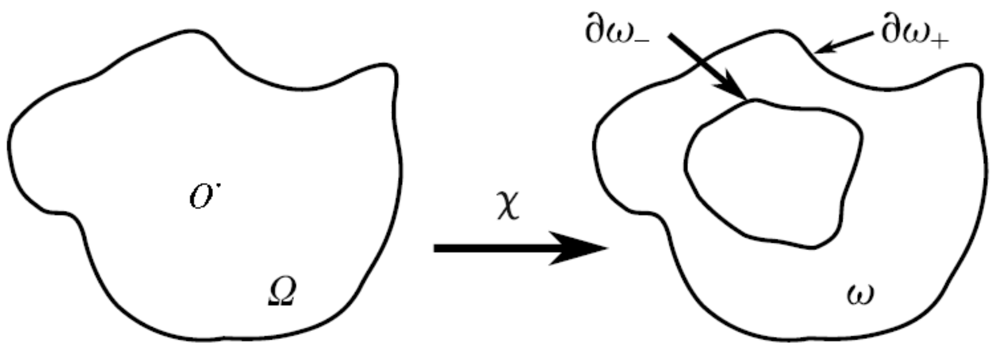
$$\rho' = \frac{a}{b}\frac{\rho}{2\cos^2\theta}\left[1+\sqrt{1-\left(\frac{b}{a}\right)^2\sin^2 2\theta}\right]$$

$$K' = \frac{b}{a}K$$

Idea works at only one angle of incidence ???

finite transformation (*deformation*)

$$\mathbf{X} \qquad\qquad \mathbf{x} = \chi(\mathbf{X})$$



$$F_{iJ} = \frac{\partial x_i}{\partial X_J}$$

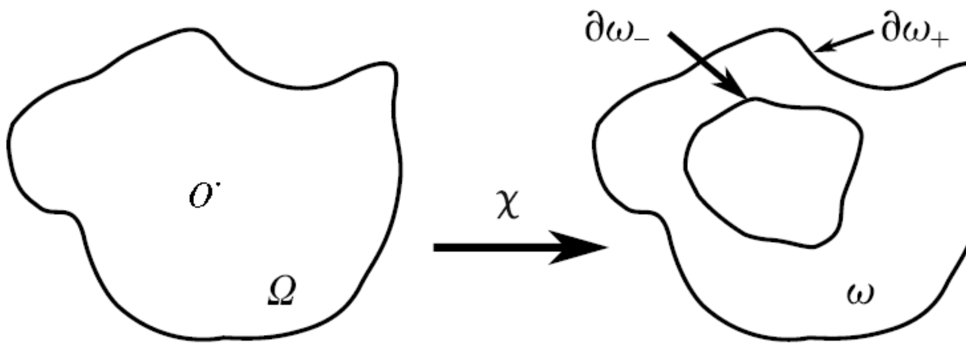
$$\mathbf{F} = \nabla_X x \qquad\qquad \mathbf{F} = \mathbf{V}\mathbf{R}$$

$$\mathbf{V}^2 = \mathbf{F}\mathbf{F}^t \qquad\qquad J = \det \mathbf{F}$$

Change of coordinates/Laplacian in new variables:

$$\nabla_X^2 p = J \,\operatorname{div}\left(J^{-1}\mathbf{V}^2 \nabla p\right)$$

transformation of the acoustic wave equation



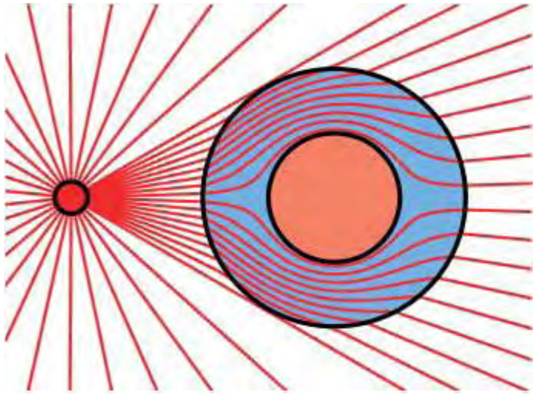
$$\nabla_X^2 p - \ddot{p} = 0 \quad \Rightarrow \quad J \,\mathrm{div}\left(J^{-1}\mathbf{V}^2 \nabla p\right) - \ddot{p} = 0$$

identical to $$K \,\mathrm{div}\left(\boldsymbol{\rho}^{-1} \nabla p\right) - \ddot{p} = 0$$

if $$K = J, \qquad \boldsymbol{\rho} = J\mathbf{V}^{-2}$$

equation transforms *if the density in the deformed (current) description is* ***anisotropic***

= idea behind **transformation acoustics**



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anisotropic inertia ?

standard acoustic wave equation $$\nabla^2 p - \ddot{p} = 0$$

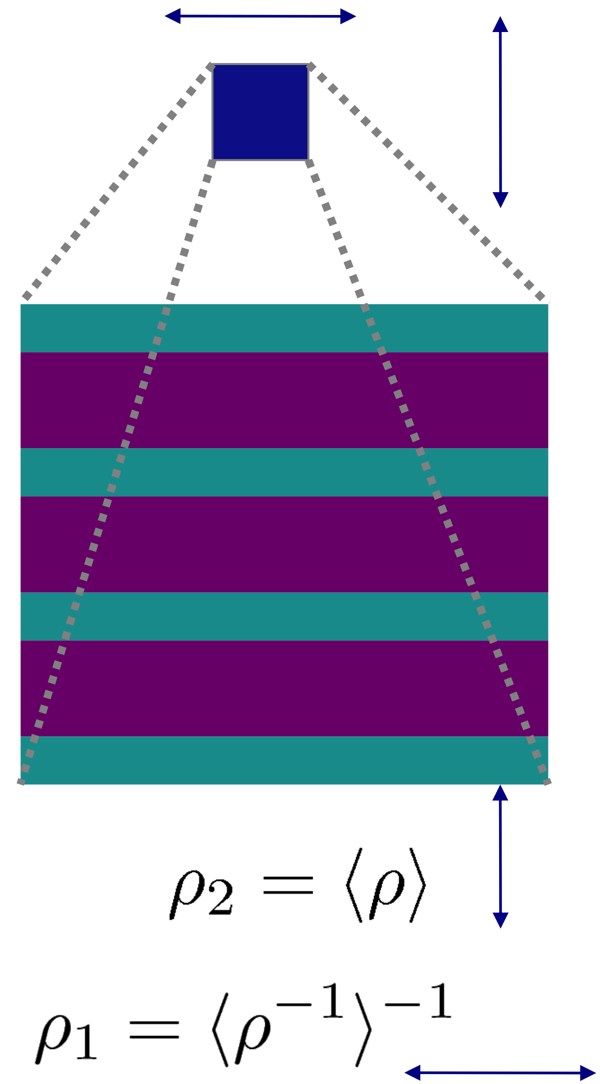
standard pressure constitutive relation $$\dot{p} = -K \operatorname{div} \mathbf{v}$$

momentum balance $$\boldsymbol{\rho}\dot{\mathbf{v}} = -\nabla p$$

with *anisotropic* inertia tensor $\boldsymbol{\rho}$

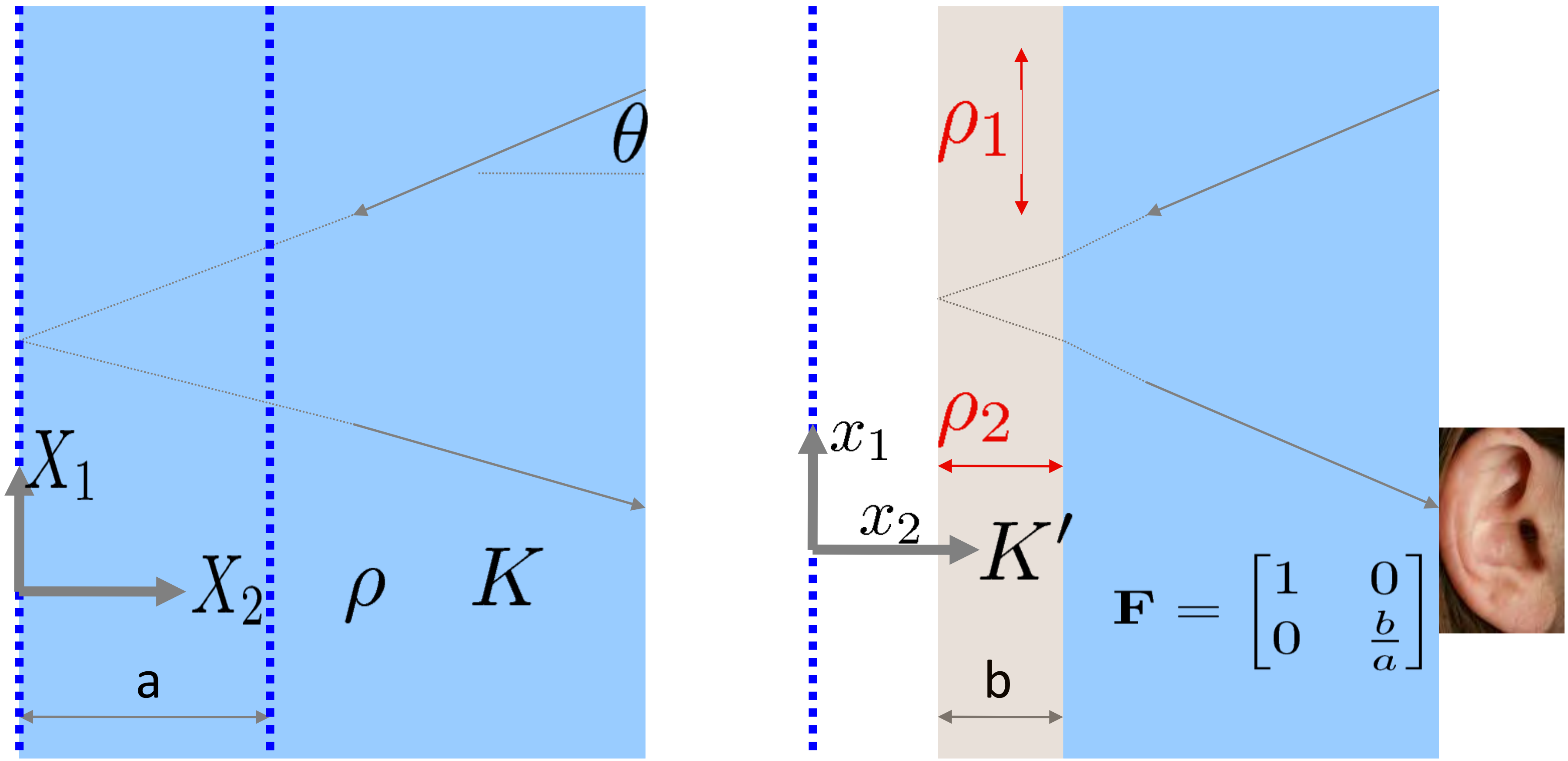
$$\Rightarrow \quad K \operatorname{div}\left(\boldsymbol{\rho}^{-1}\nabla p\right) - \ddot{p} = 0$$

modified acoustic wave equation



e.g. layered fluid exhibits anisotropic inertia

2D mirage using anisotropic inertia



$$K = JK, \qquad \boldsymbol{\rho} = J\mathbf{V}^{-2}\rho$$

$$\Rightarrow \quad K' = \frac{b}{a}K$$

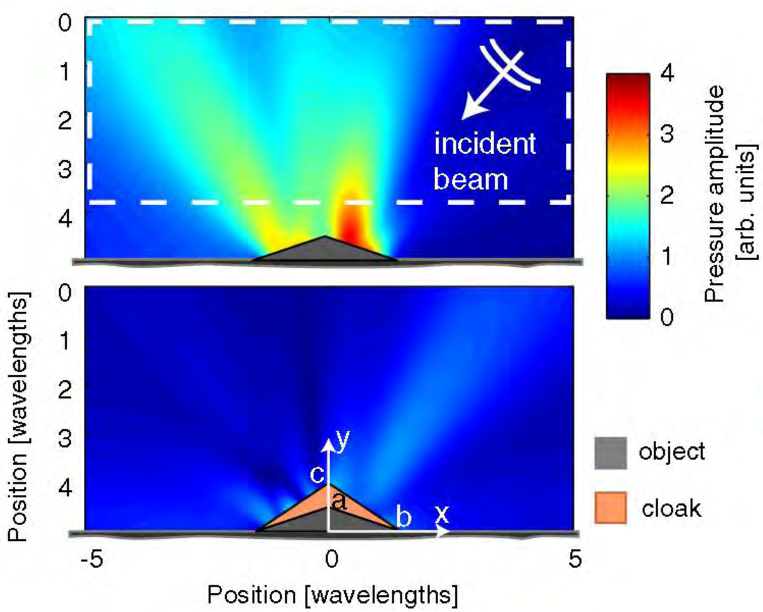
$$J = \det \mathbf{F} \qquad \mathbf{V}^2 = \mathbf{F}\mathbf{F}^t$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j}$$

$$\rho_1 = \frac{b}{a}\rho, \qquad \rho_2 = \frac{a}{b}\rho$$

works for **all** angles of incidence

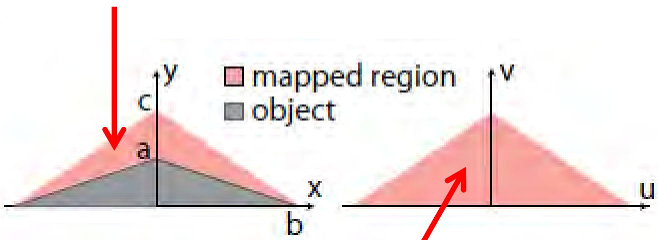
Acoustic **carpet cloak** in air



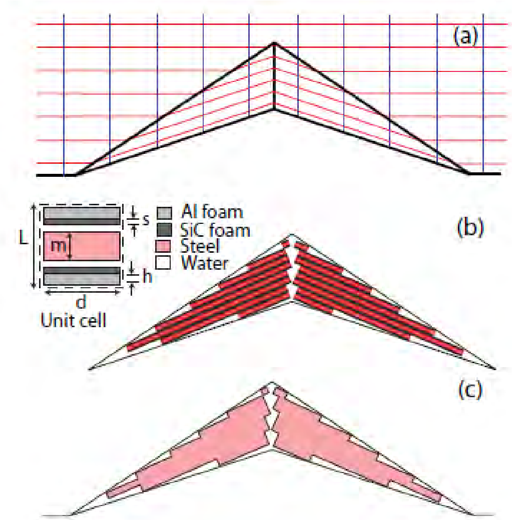
Popa, Zigoneanu, Cummer (PRL June 2011)

How?

physical region is mapped



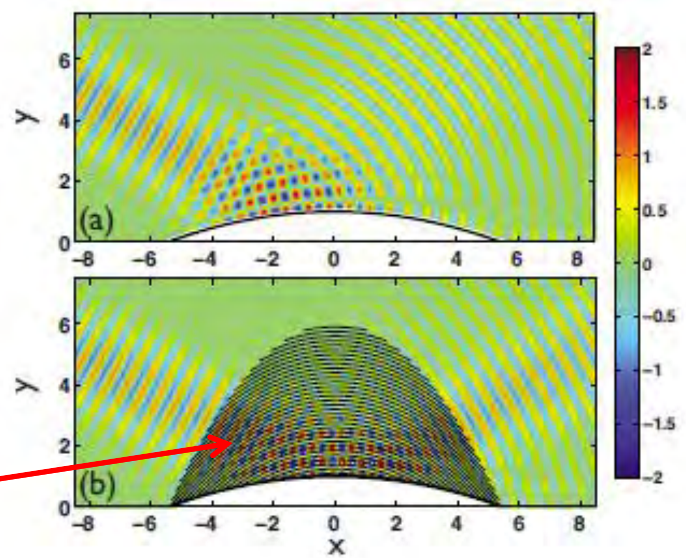
into larger **virtual** one



layered scaffold of heavy plates

-has effect of slowing & curving the wave so that it *appears* to reflect from the flat surface

- curved 2D mirage device
- carpet cloak in water using curved steel plates

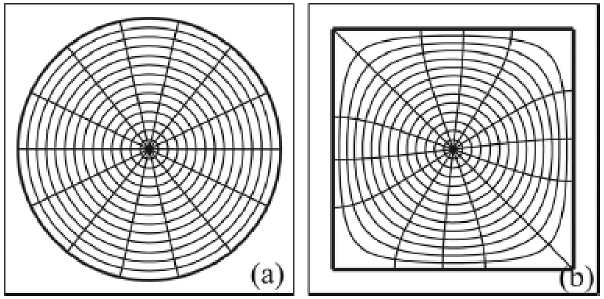


Liang & Li (APL 2011)

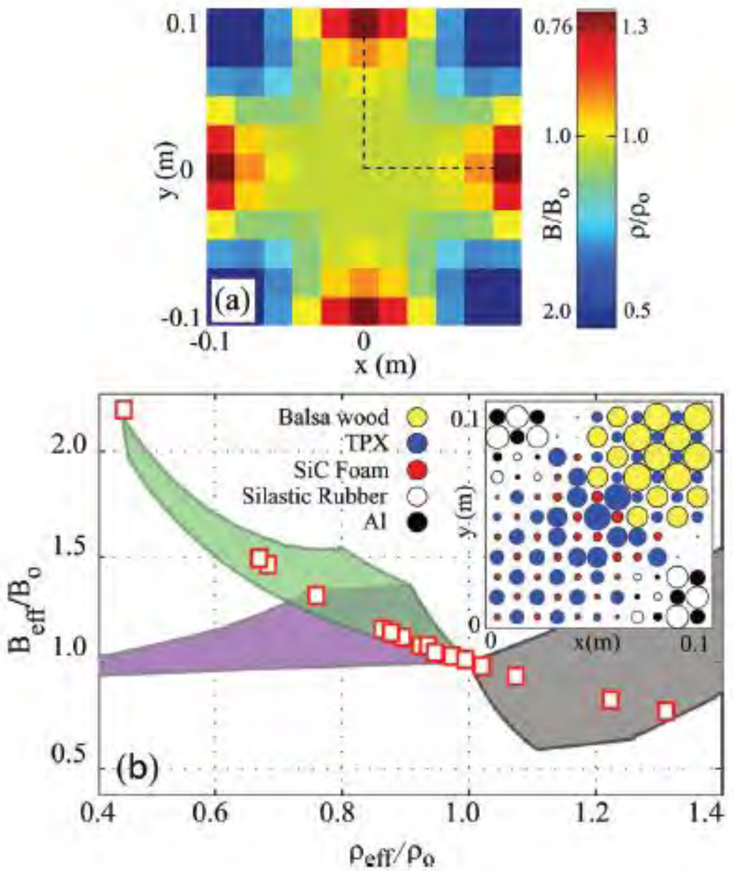
using transformation acoustics to design wave control devices

example: cylindrical-to-plane wave lens

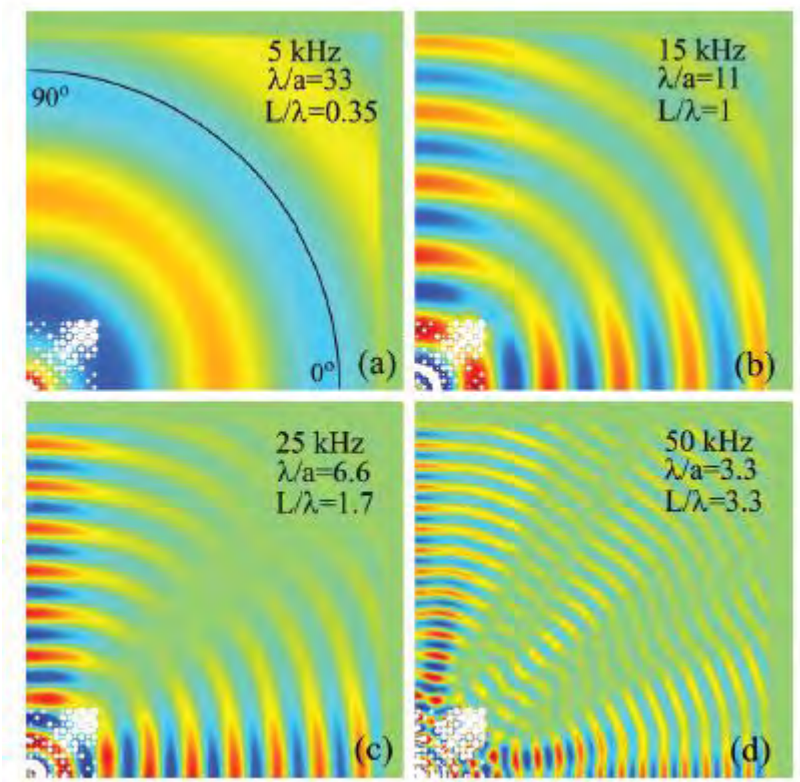
Layman et al. (APL 2011 doi: 10.1063/1.3652914)



virtual physical



range of materials used



cylindrical source radiates as plane waves

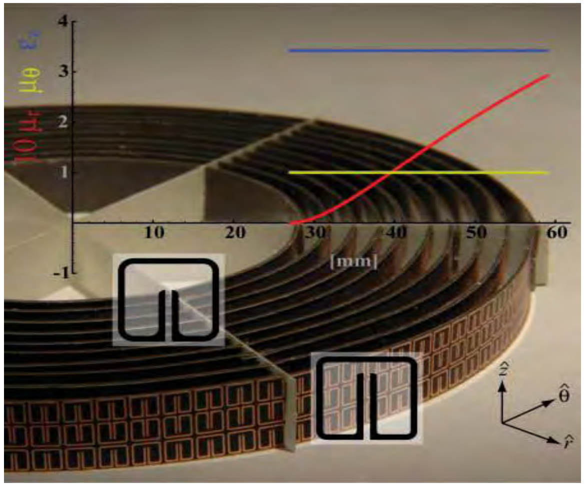
anisotropy is a big part of cloaking

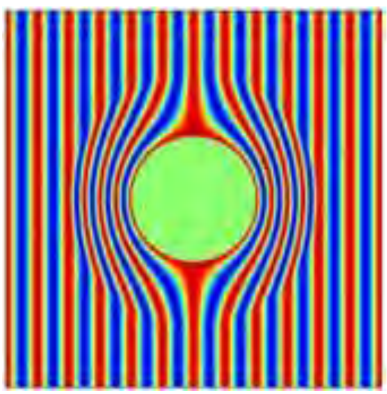
$$\varepsilon_r = \mu_r = \frac{r-a}{r}, \qquad a < r < b$$

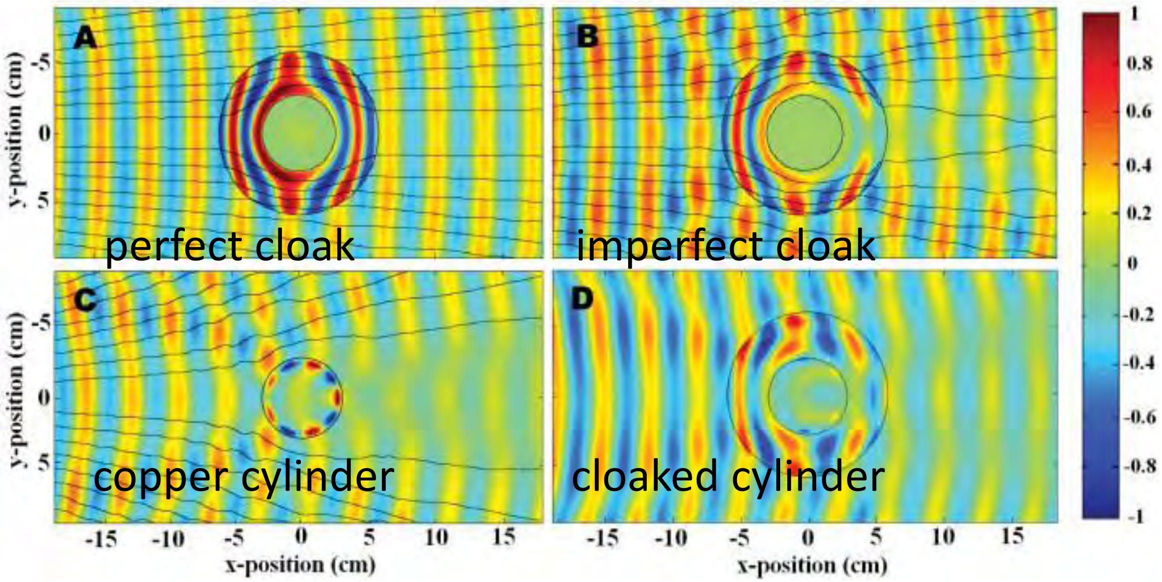
$$\varepsilon_\theta = \mu_\theta = \frac{r}{r-a}$$

$$\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}$$

extremely *anisotropic*



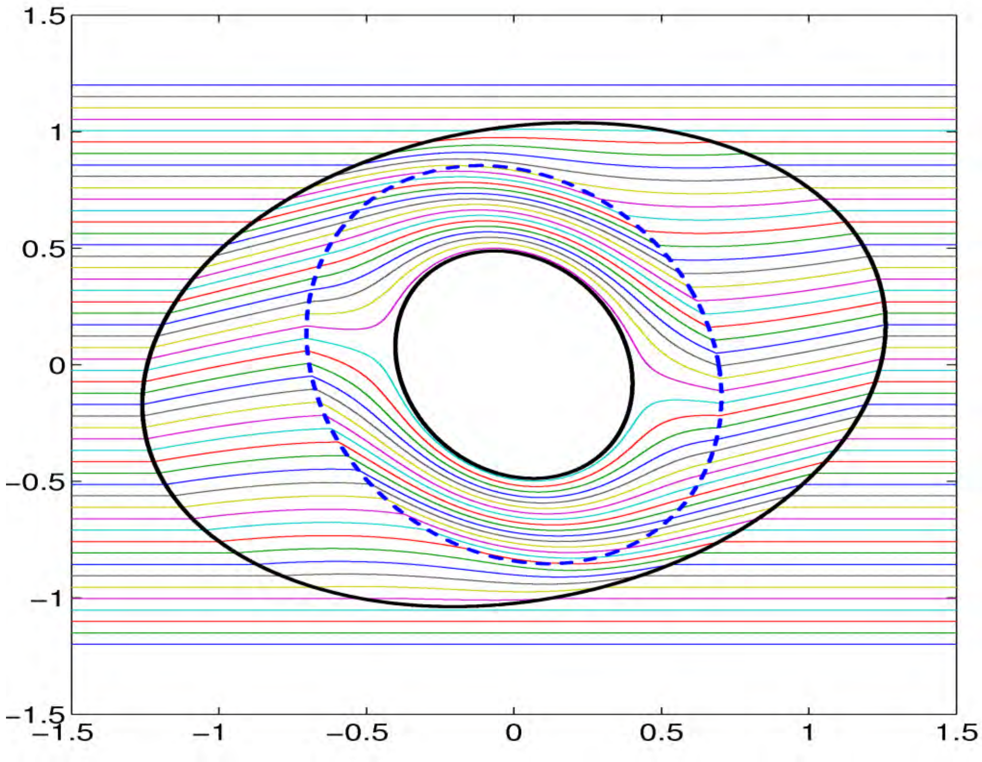




Metamaterial Electromagnetic Cloak at Microwave Frequencies Schurig, et al., *Science*, 2006

aside: strange effects in cloaking - rays in a cloak

rays = deformed straight lines under transformation **F**



the wavefront bifurcates

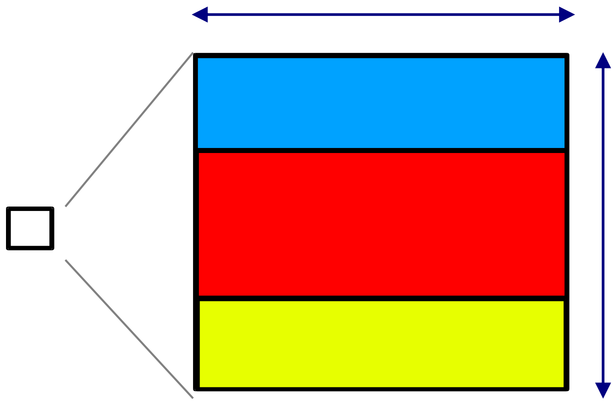
the line through the singular point becomes a *split ray*

the “uncaustic”

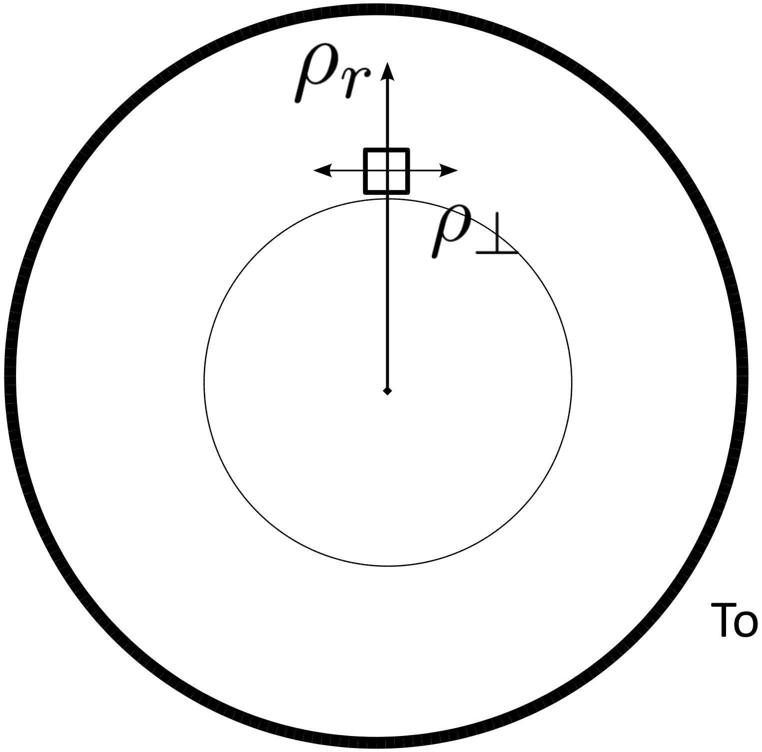
ray bundle density $\longrightarrow 0$

cylindrical anisotropic inertia

$$\rho_\perp = \langle \rho^{-1} \rangle^{-1}$$



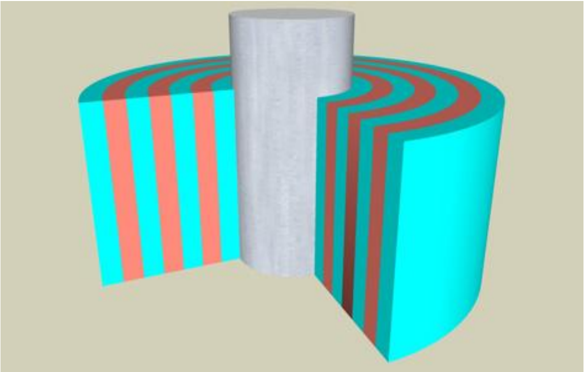
$$\rho_r = \langle \rho \rangle$$



Momentum Balance

$$\boldsymbol{\rho} \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p$$

$$[\boldsymbol{\rho}] = \begin{bmatrix} \rho_r & 0 \\ 0 & \rho_\perp \end{bmatrix}$$



Torrent and Sanchez-Dehesa **(2008)**

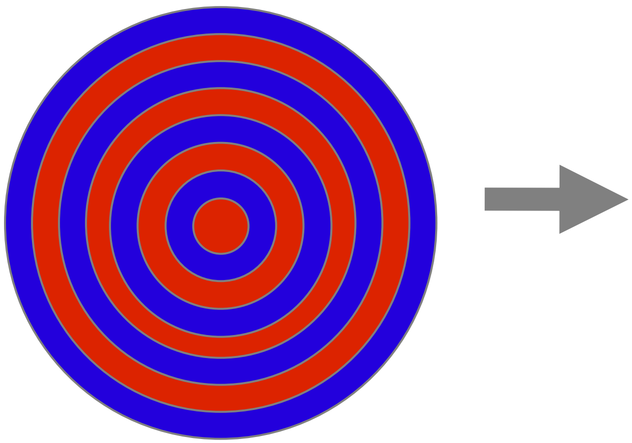
homogenized cylinders with anisotropic inertia

ρ_1, c_1 (blue)

ρ_2, c_2 (red)

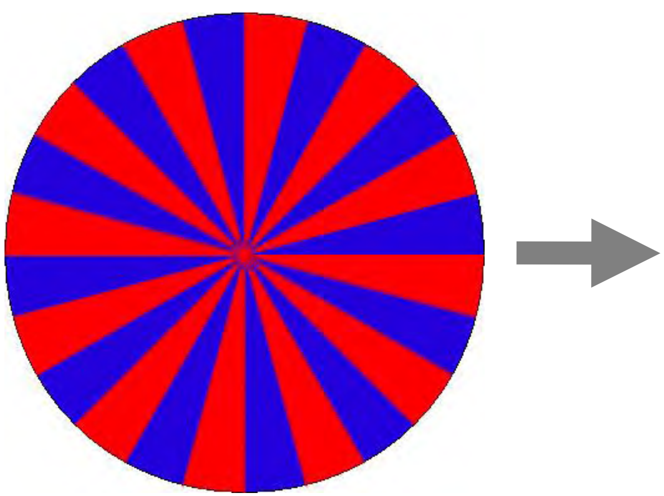
effective properties $\rho_r, \rho_\theta, C_{eff} = \langle C \rangle$

$$C = K^{-1}$$



$$\rho_r = \langle \rho \rangle$$

$$\rho_\theta = \langle \rho^{-1} \rangle^{-1}$$

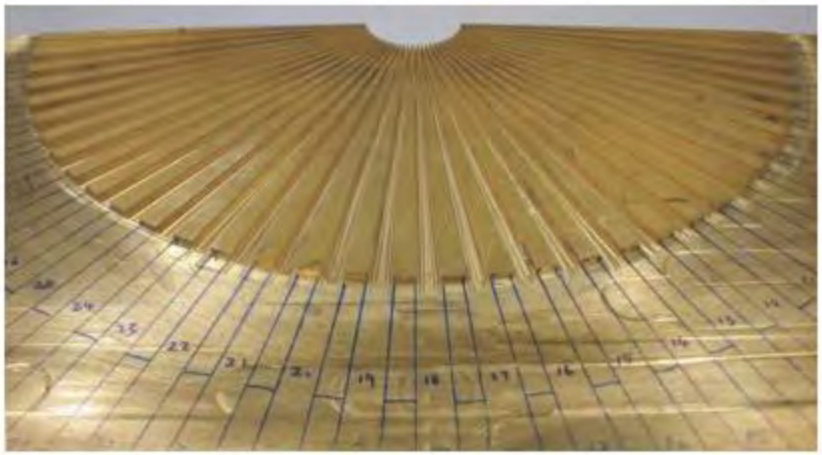


$$\rho_r = \langle \rho^{-1} \rangle^{-1}$$

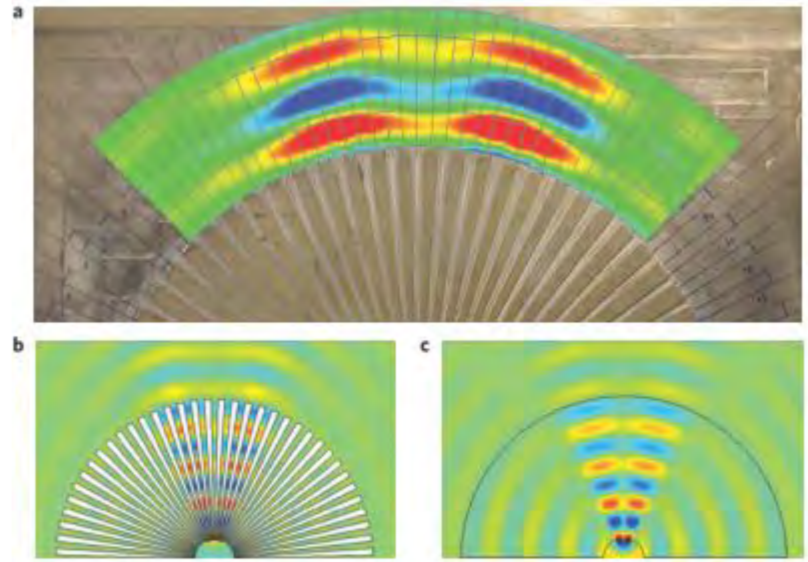
$$\rho_\theta = \langle \rho \rangle$$

application: acoustic hyperlens

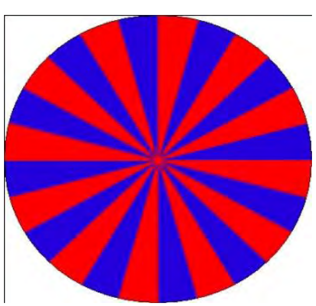
images sub-wavelength sources



Li et al. (Nature Materials 2009)
doi: 10.1038/nmat2561

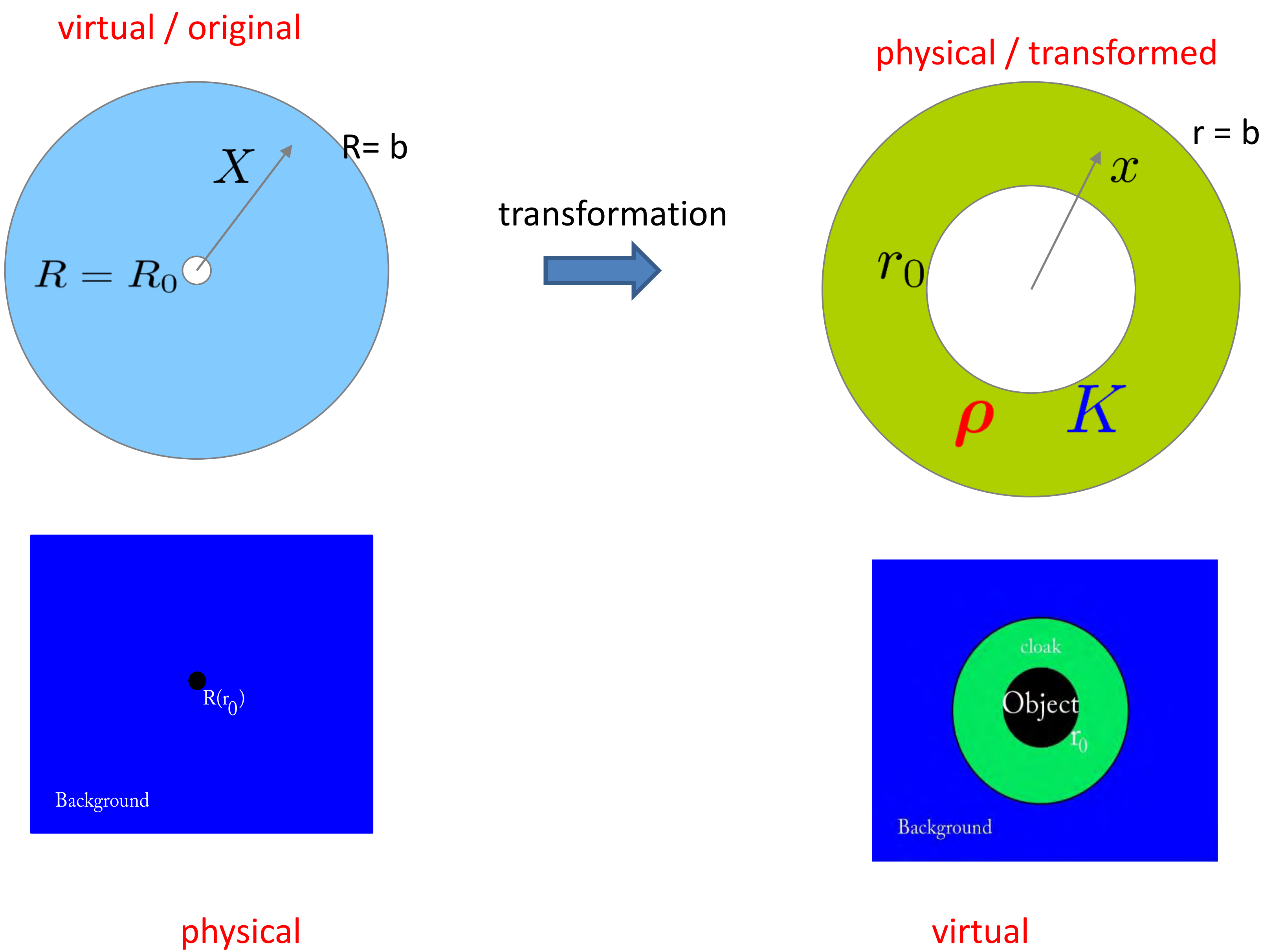


sources

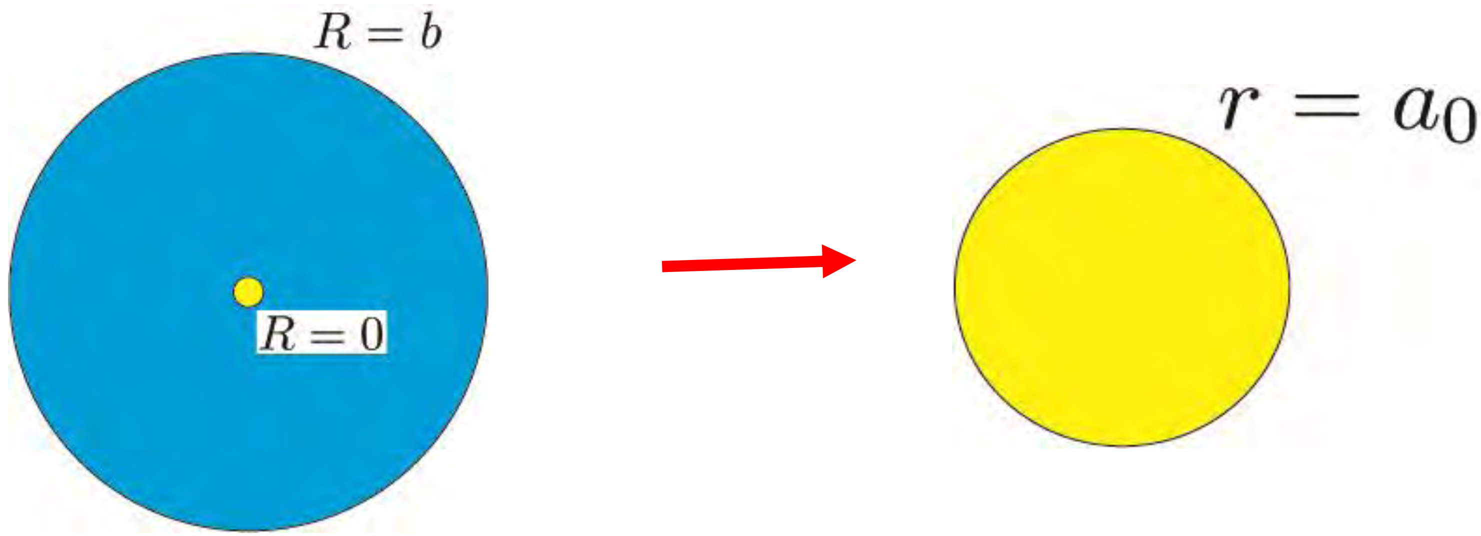


$$\rho_r = \langle \rho^{-1} \rangle^{-1}$$
$$\rho_\theta = \langle \rho \rangle$$

cloaking : metafluid + hole

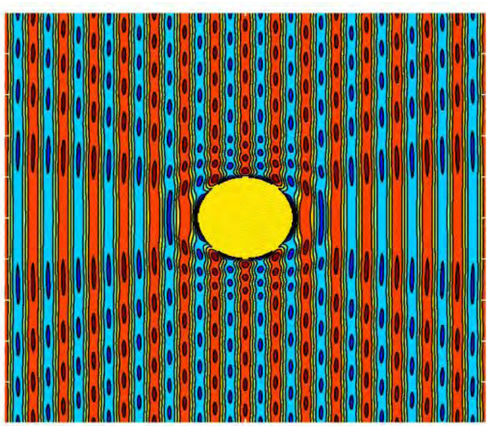


radially symmetric cloak

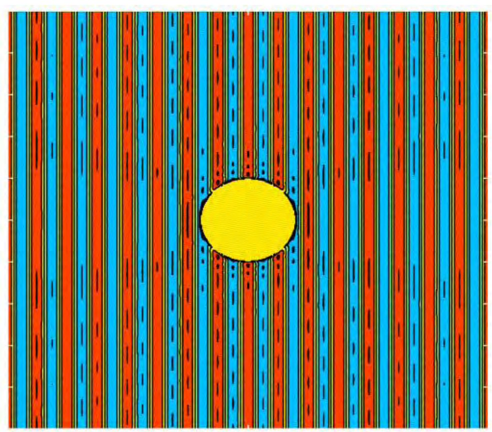


blow up R=0 into ball of radius a_0 $\quad \mathbf{X} = f(r)\hat{\mathbf{x}} \quad a_0 \le r \le b \qquad f(a_0) = 0$

example transformation: $f(r) = \dfrac{r - a_0}{b - a_0}\, b$



imperfect or “near cloak”



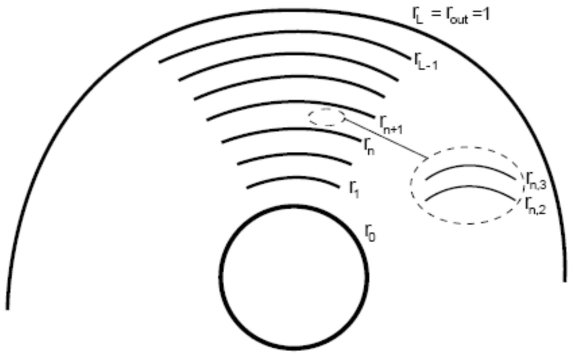
perfect (almost)

The *perfect* cloak has inner boundary equal to the image of a point

In a *near* cloak it is a small hole

three-fluid inertial cloak (Norris and Nagy, JASA 2010)

$$\begin{pmatrix} 1 & 1 & 1 \\ \rho_1 & \rho_2 & \rho_3 \\ \rho_1^{-1} & \rho_2^{-1} & \rho_3^{-1} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \rho_r \\ \rho_\perp^{-1} \end{pmatrix}$$



$$\Rightarrow \qquad \boldsymbol{\phi} = \boldsymbol{f}_0 + \rho_r \boldsymbol{f}_1 + \rho_\perp^{-1} \boldsymbol{f}_2$$

effective compressibility: $C_* = \alpha + \beta_1 \rho_r + \beta_2 {\rho_\perp}^{-1}$

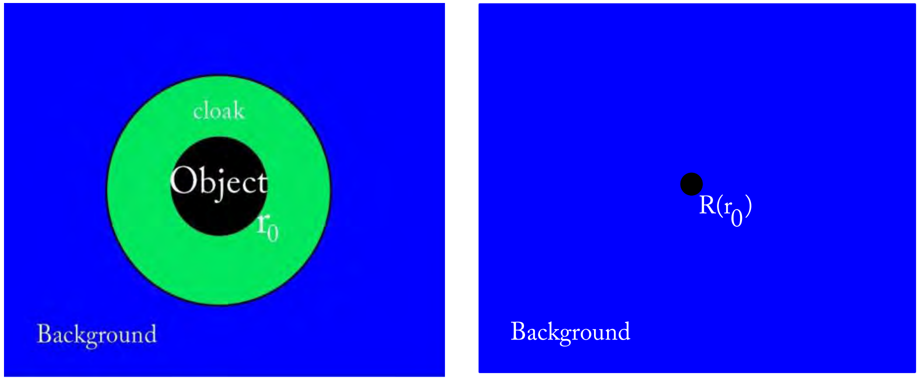
$$C_* = R'\left(\frac{R}{r}\right)^{d-1}, \quad \rho_r = R'\left(\frac{r}{R}\right)^{d-1}, \quad {\rho_\perp}^{-1} = R'\left(\frac{R}{r}\right)^{d-3}$$

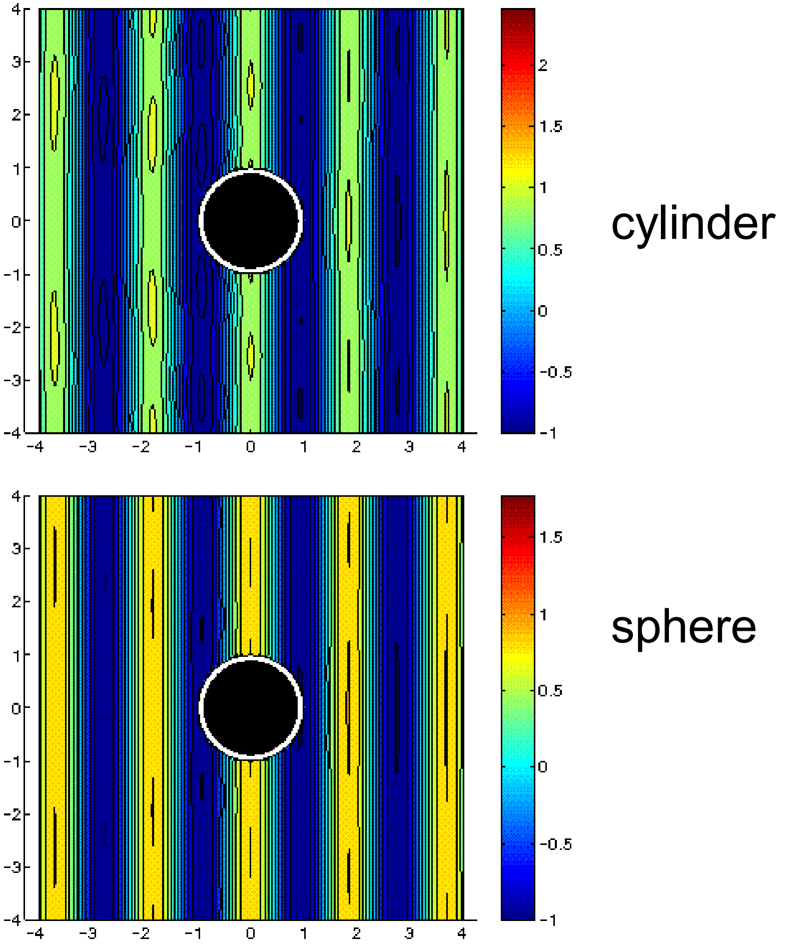
ODE for R(r) $\Rightarrow$ explicit transformation r → R(r)

Bottom line: **can achieve exact transformation using only 3 distinct fluids mixed in different proportions**

example: rigid scatterer with 3-fluid near-cloak

- can reduce scattering cross-section to less than 1%
- requires one very heavy fluid, one very light
- total mass of device is LARGE

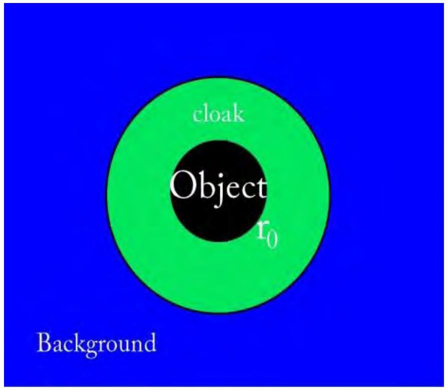


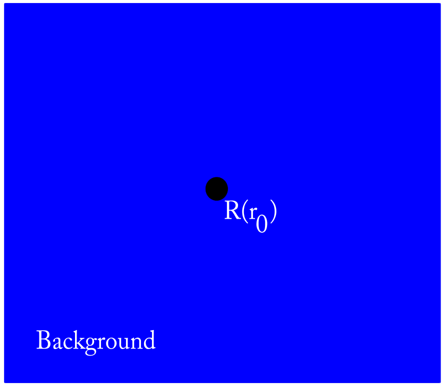


- BUT – the types of fluid required are not available
  e.g. huge density + huge compressibility

$$kr_0 = 3$$

near cloaks made of inertial metafluids have *very large mass*





total mass of 3D near-cloak

$$m = \frac{3}{b^3 - r_0^3}\left(\frac{r_0^4}{R(r_0)} - \frac{b^4}{R(b)}\right) + \text{finite}$$

perfect cloak: $R(r_0) = 0$

The perfect cloak has infinite mass

– regardless of the transformation (2D or 3D, radially symmetric or not)

???

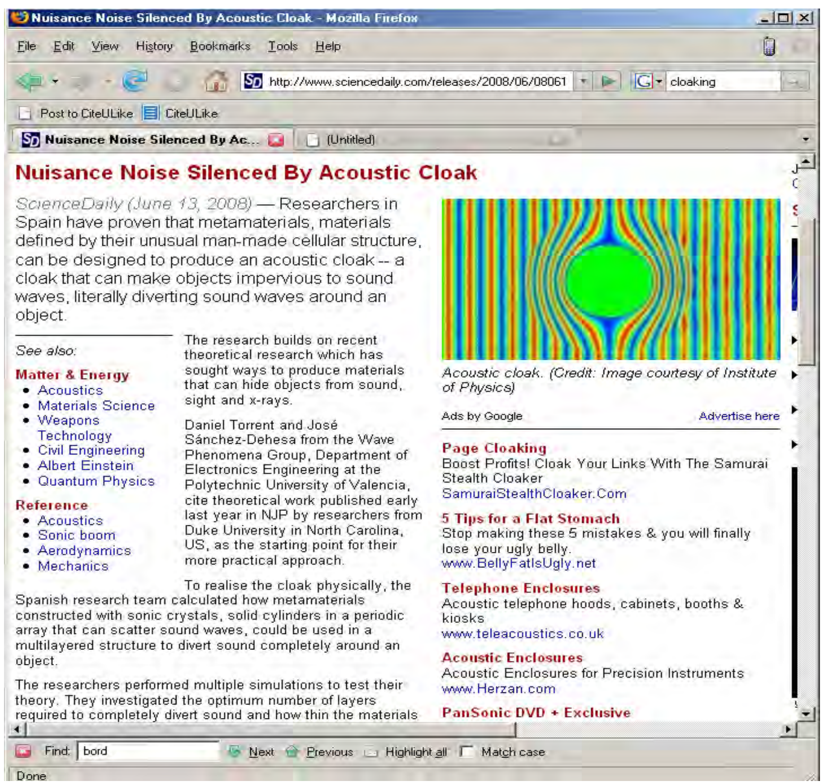
How to make a cloak?

Acoustic ``Invisibility" Cloaks Possible, Study Says	*National Geographic* 1/08
How to Make a Submarine Disappear	*Science/AAAS* 1/2008
Acoustic metamaterials: Silence all around	*Nature/China* 11/07
How to Build an Acoustic Invisibility Cloak	*New Scientist* 8/07

etc.

All inertial cloaks

- infinitely massive



Nuisance Noise Silenced By Acoustic Cloak - Mozilla Firefox

Nuisance Noise Silenced By Acoustic Cloak

ScienceDaily (June 13, 2008) — Researchers in Spain have proven that metamaterials, materials defined by their unusual man-made cellular structure, can be designed to produce an acoustic cloak -- a cloak that can make objects impervious to sound waves, literally diverting sound waves around an object.

Acoustic cloak. (Credit: Image courtesy of Institute of Physics)

See also:

Matter & Energy
- Acoustics
- Materials Science
- Weapons Technology
- Civil Engineering
- Albert Einstein
- Quantum Physics

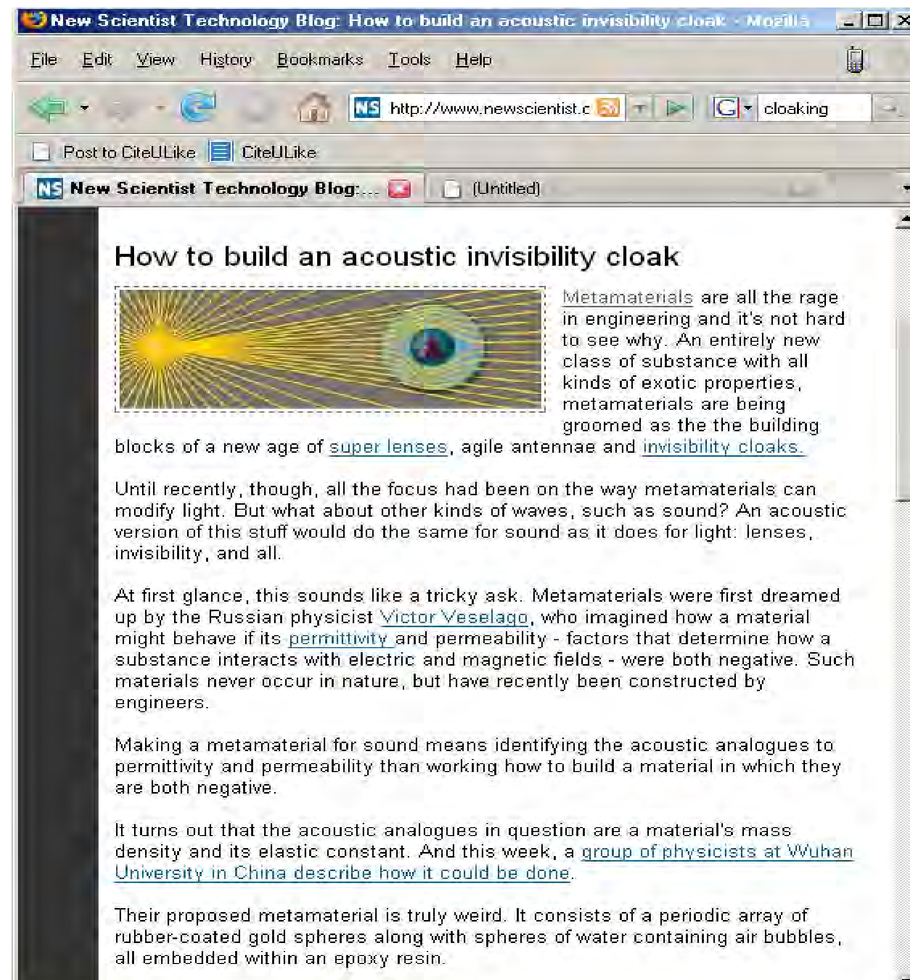
Reference
- Acoustics
- Sonic boom
- Aerodynamics
- Mechanics

The research builds on recent theoretical research which has sought ways to produce materials that can hide objects from sound, sight and x-rays.

Daniel Torrent and José Sánchez-Dehesa from the Wave Phenomena Group, Department of Electronics Engineering at the Polytechnic University of Valencia, cite theoretical work published early last year in NJP by researchers from Duke University in North Carolina, US, as the starting point for their more practical approach.

To realise the cloak physically, the Spanish research team calculated how metamaterials constructed with sonic crystals, solid cylinders in a periodic array that can scatter sound waves, could be used in a multilayered structure to divert sound completely around an object.

The researchers performed multiple simulations to test their theory. They investigated the optimum number of layers required to completely divert sound and how thin the materials



New Scientist Technology Blog: How to build an acoustic invisibility cloak - Mozilla

How to build an acoustic invisibility cloak

Metamaterials are all the rage in engineering and it's not hard to see why. An entirely new class of substance with all kinds of exotic properties, metamaterials are being groomed as the the building blocks of a new age of super lenses, agile antennae and invisibility cloaks.

Until recently, though, all the focus had been on the way metamaterials can modify light. But what about other kinds of waves, such as sound? An acoustic version of this stuff would do the same for sound as it does for light: lenses, invisibility, and all.

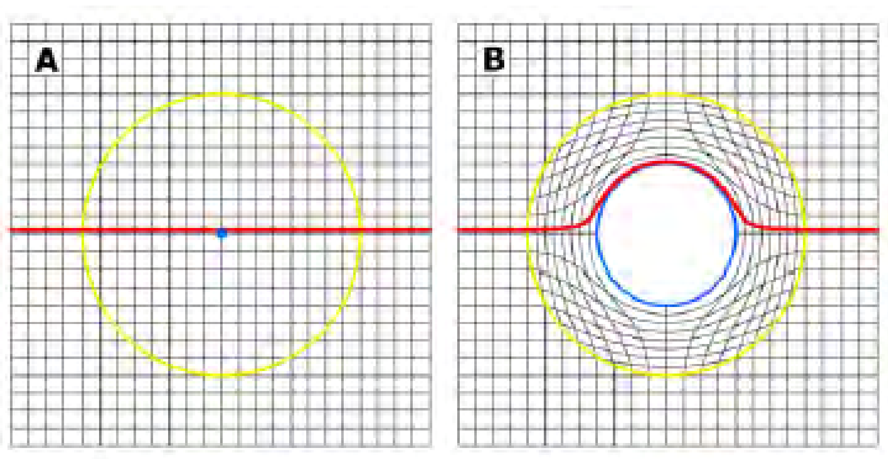
At first glance, this sounds like a tricky ask. Metamaterials were first dreamed up by the Russian physicist Victor Veselago, who imagined how a material might behave if its permittivity and permeability - factors that determine how a substance interacts with electric and magnetic fields - were both negative. Such materials never occur in nature, but have recently been constructed by engineers.

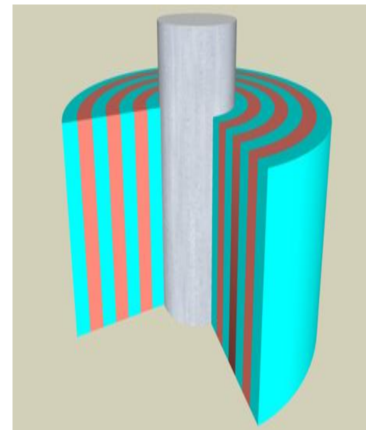
Making a metamaterial for sound means identifying the acoustic analogues to permittivity and permeability than working how to build a material in which they are both negative.

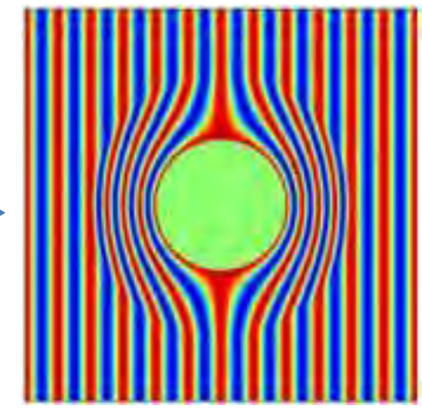
It turns out that the acoustic analogues in question are a material's mass density and its elastic constant. And this week, a group of physicists at Wuhan University in China describe how it could be done.

Their proposed metamaterial is truly weird. It consists of a periodic array of rubber-coated gold spheres along with spheres of water containing air bubbles, all embedded within an epoxy resin.

Path to acoustic cloaking?





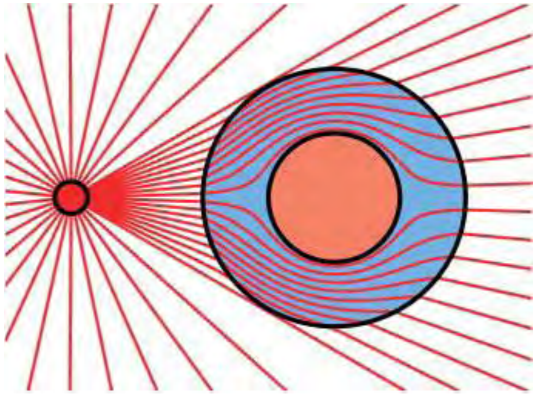


Transformation -> radially anisotropic density -> cloak -> zero scattering

Making a fluid with strong density anisotropy is **difficult/impossible**.

Other issues: total mass required is very large

Why density? **Why not bulk modulus?**



Introduction: metamaterials

Phononic crystals – engineering the band gap

waves in periodic systems

examples of phononic crystals

Acoustic cloaking – engineering the impossible

transformation acoustics

1D, 2D, cylindrical

inertial materials

Pentamode materials

cloaking elastic waves

Metatheory for metamaterials?

cloaking

pentamode materials

water as an elastic “solid”

elastic equation of motion $\mathrm{div}\boldsymbol{\sigma} = \boldsymbol{\rho}\ddot{\mathrm{u}}$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad \rightarrow \quad \boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}$$

Kelvin (1856) $$C_{ijkl} = \sum_{\alpha=1}^{6} K_\alpha P_{ij}^\alpha P_{kl}^\alpha \quad \rightarrow \quad \mathbf{C} = \sum_{\alpha=1}^{6} K_\alpha \mathbf{P}^\alpha \otimes \mathbf{P}^\alpha$$

Acoustics $$C_{ijkl} = K\,\delta_{ij}\delta_{kl} \quad \rightarrow \quad \mathbf{C} = K\,\mathbf{I} \otimes \mathbf{I}$$

Five of the eigen-stiffnesses are zero. Water is a **pentamodal elastic material**

(Milton & Cherkaev 1994) five (*penta)* easy modes

general pentamode form of stiffness:
S is divergence free

$$\boldsymbol{C} = K\boldsymbol{S} \otimes \boldsymbol{S}$$

pentamode material = transformed acoustic medium

constitutive relation	momentum balance
$\boldsymbol{\sigma} = K\,\mathrm{tr}(\mathbf{S}\boldsymbol{\varepsilon})\,\mathbf{S}$	$\boldsymbol{\rho}\dot{\mathbf{v}} = \mathrm{div}\,\boldsymbol{\sigma}$

rewrite in 'acoustic' form:

$$\boldsymbol{\sigma} = -p\mathbf{S}$$

$$\dot{p} = -K\mathbf{S} : \nabla\mathbf{v}$$

pseudo-pressure p(x,t)

Use the PM *compatibility* condition $\mathrm{div}\,\mathbf{S} = 0$

wave equation for the pseudo-pressure

$$K\mathbf{S} : \nabla\left(\boldsymbol{\rho}^{-1}\mathbf{S}\nabla p\right) - \ddot{p} = 0$$

(was $K\,\mathrm{div}\left(\boldsymbol{\rho}^{-1}\nabla p\right) - \ddot{p} = 0$)

Can transform back to acoustic eq in X for *arbitrary* ***S*** as long as it satisfies $\mathrm{div}\,\mathbf{S} = 0$

why can we have different types of transformed materials?

started with $$\nabla_X^2 p - \ddot{p} = 0$$

transformed coordinates $\Longrightarrow$ $$J \ \mathrm{div}\left(J^{-1}\mathbf{V}^2\nabla p\right) - \ddot{p} = 0$$

interpreted as eq. of acoustic fluid with anisotropic inertia $K = J, \qquad \boldsymbol{\rho} = J\mathbf{V}^{-2}$

but we did not change the meaning of the pressure (or particle displacement)

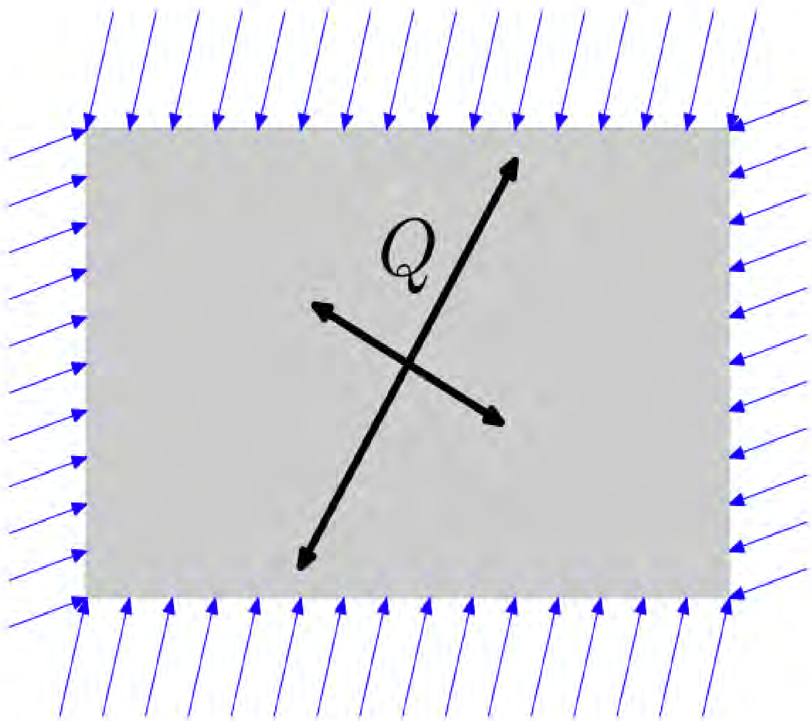
relax this and redefine a "transformed" displacement

U = A u

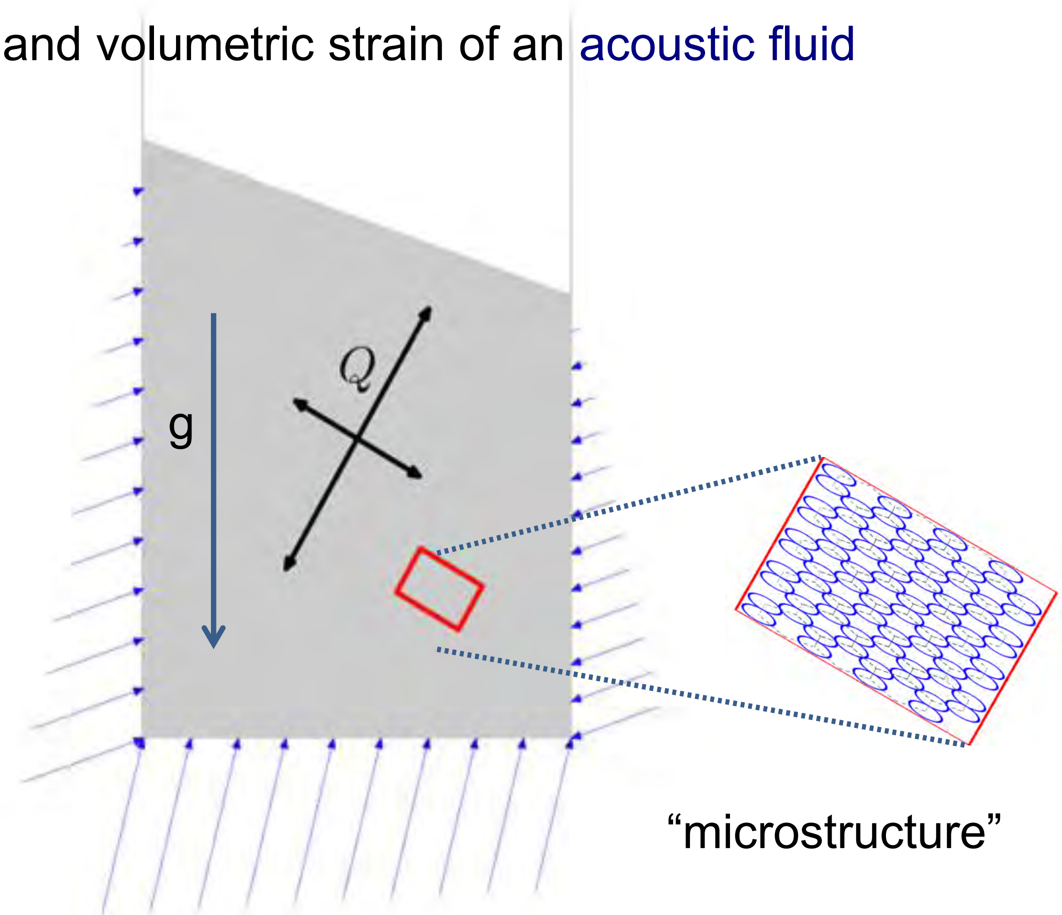
then **A** is related to appearance of the matrix **S** in the pentamode material mechanical behavior of **pentamode materials** $C_{ijkl} = K\, Q_{ij} Q_{kl}$

a single type of stress (and strain)

- generalize hydrostatic stress and volumetric strain of an acoustic fluid



Static equilibrium of a block



... under gravity

summary of pentamode material transformation

The metafluid is any material of the form *

$$K = J, \qquad \mathbf{C} = K\mathbf{S} \otimes \mathbf{S}, \qquad \boldsymbol{\rho} = J\mathbf{S}\mathbf{V}^{-2}\mathbf{S},$$

(Norris '08, '09)

- **J** (Jacobian) and **V** (metric) are defined by the (arbitrary) transformation
- **S** is *any* definite symmetric tensor satisfying $\operatorname{div} \mathbf{S} = 0$

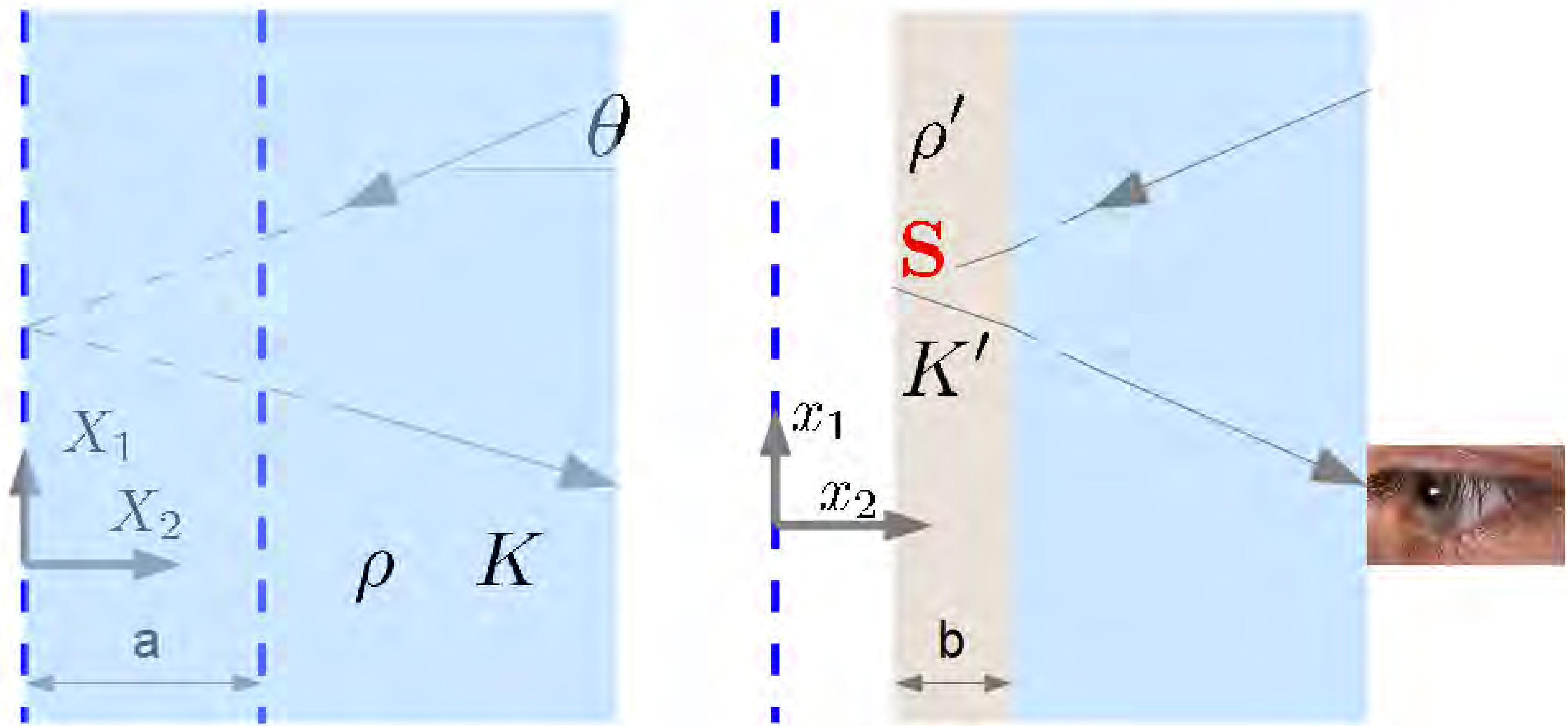
anisotropic inertia is the special case **S** = **I**

S introduces additional degrees of freedom, non-uniqueness (unlike EM)

1) most general type of acoustic metafluid is **PM** with anisotropic inertia

2) can get *isotropic density* by choosing **S** to make it so

*it can be even more general: **S** does *not* have to be symmetric - leading to nonsymmetric stress mirages using *pentamode* material



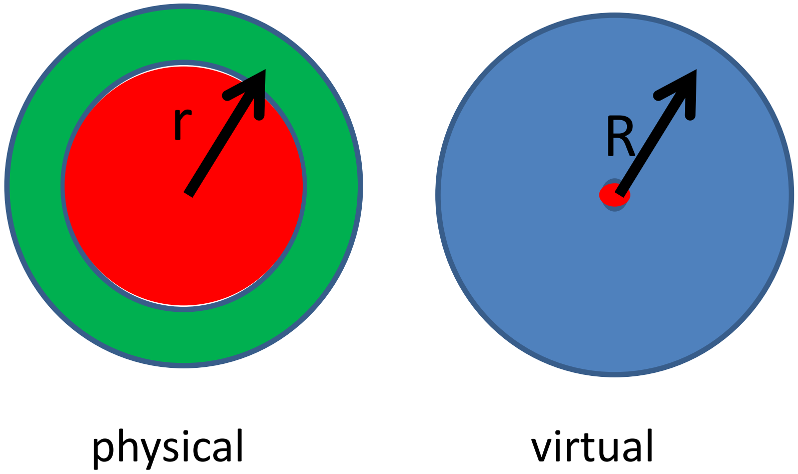
$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{b}{a} \end{bmatrix} \qquad \boldsymbol{\rho}' = J^{-1}\rho\mathbf{I}, \qquad K' = JK, \qquad \mathbf{S} = J^{-1}\mathbf{V},$$

$$\Longrightarrow \qquad K' = \frac{b}{a}K \qquad \rho' = \frac{a}{b}\rho \qquad \mathbf{S} = \begin{bmatrix} \frac{a}{b} & 0 \\ 0 & 1 \end{bmatrix}$$

Isotropic density with total mass conserved
PM supports biaxial stress

pentamode material transformation

radially symmetric transformation



$$\rho = R'\left(\frac{R}{r}\right)^{d-1}, \quad K_r = \frac{1}{R'}\left(\frac{R}{r}\right)^{d-1}, \quad K_\perp = R'\left(\frac{R}{r}\right)^{d-3}$$

- looks a lot like the inertial anisotropy transformation:

$$\frac{1}{K} = R'\left(\frac{R}{r}\right)^{d-1}, \quad \frac{1}{\rho_r} = \frac{1}{R'}\left(\frac{R}{r}\right)^{d-1}, \quad \frac{1}{\rho_\perp} = R'\left(\frac{R}{r}\right)^{d-3}$$

- gives the same wave steering effects

- but the **mechanics** is completely different

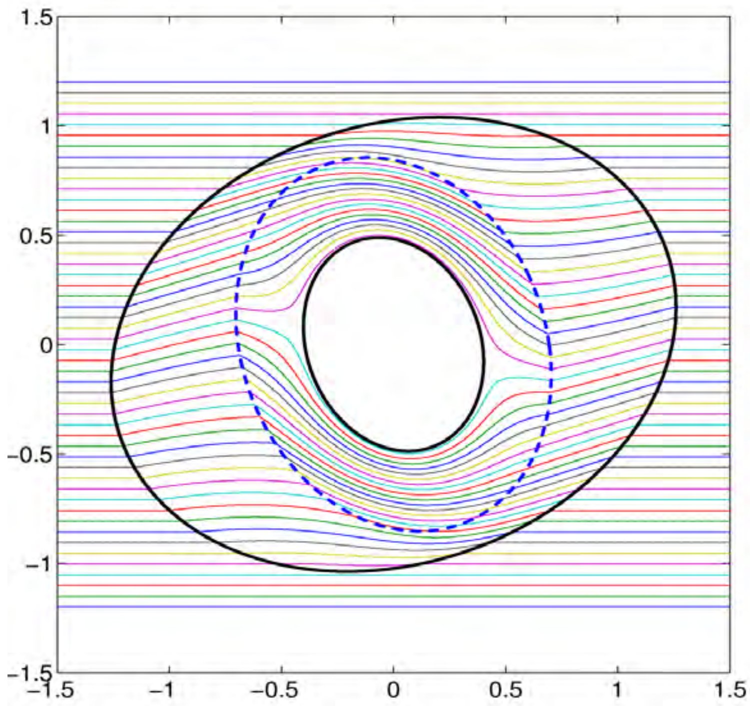
transformed PM has isotropic inertia

freedom to choose **S** means the cloak material has isotropic inertia
e.g. for any radially symmetric transformation

Important PM property - like 1D (used later):

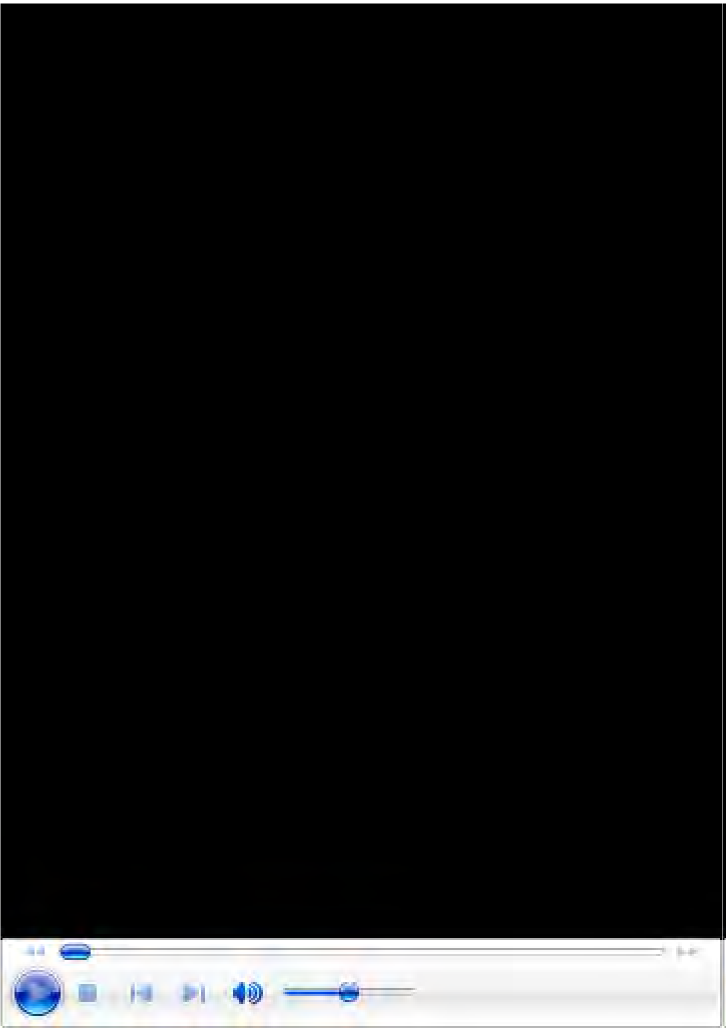
$$\int_{\Omega} dv J^{-1} \rho = \int_{\Omega_0} dv_0 \rho_0 = m(\Omega_0)$$

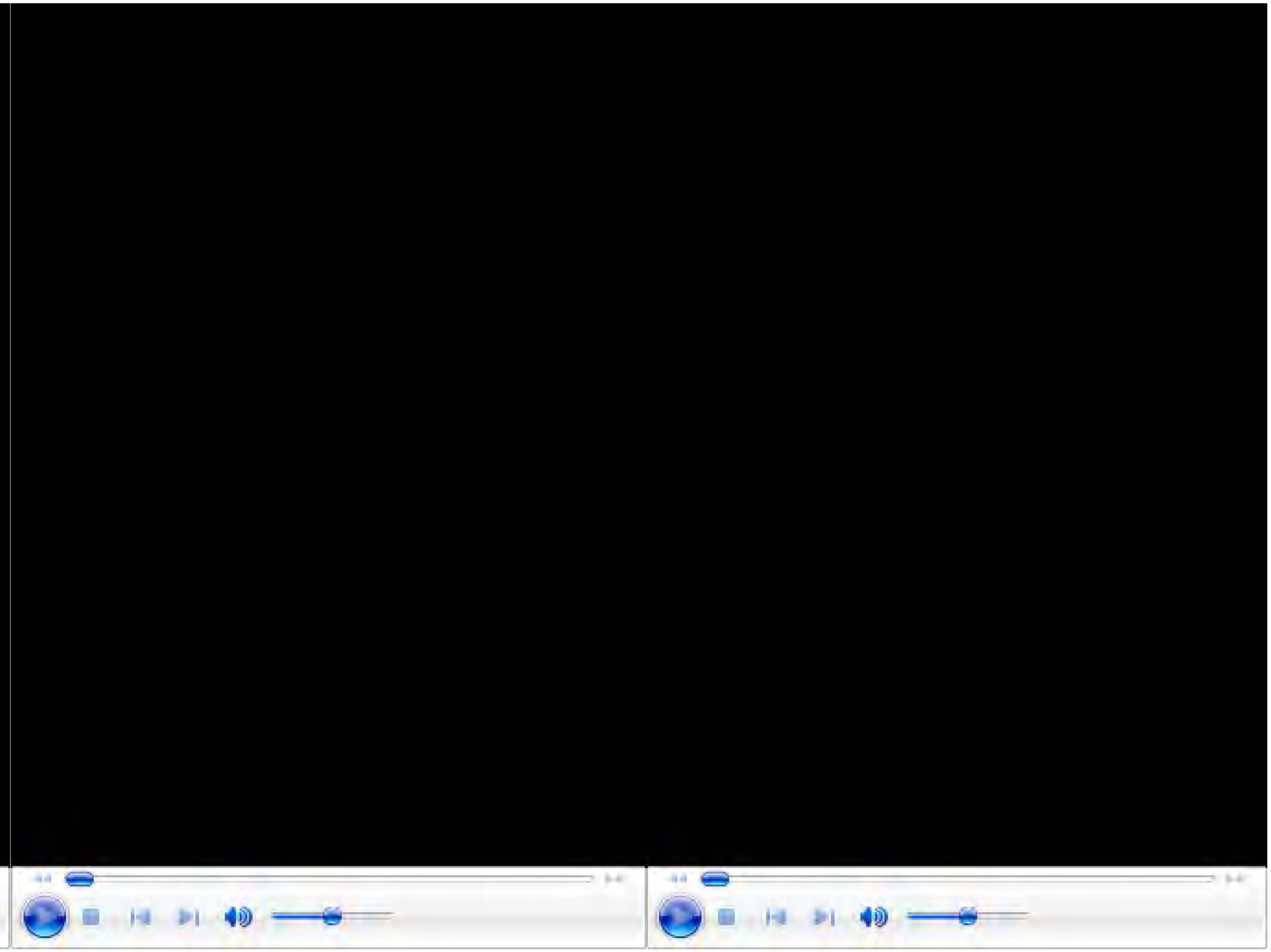
The total mass of the cloak is conserved under the transformation



a non-radially symmetric cloak
with **isotropic** density
(& finite mass of course)

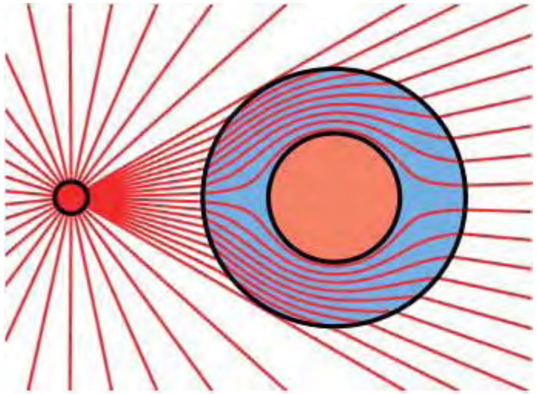
simulation: acoustic scattering from a steel sphere





no cloak

layered pentamode cloak



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inertial materials

Pentamode materials - design

cloaking elastic waves

Metatheory for metamaterials?

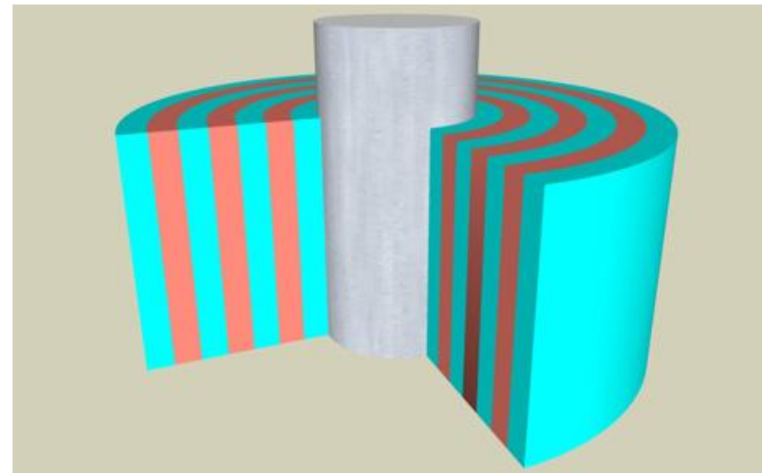
design of

pentamode materials

possible designs of acoustic cloaking devices

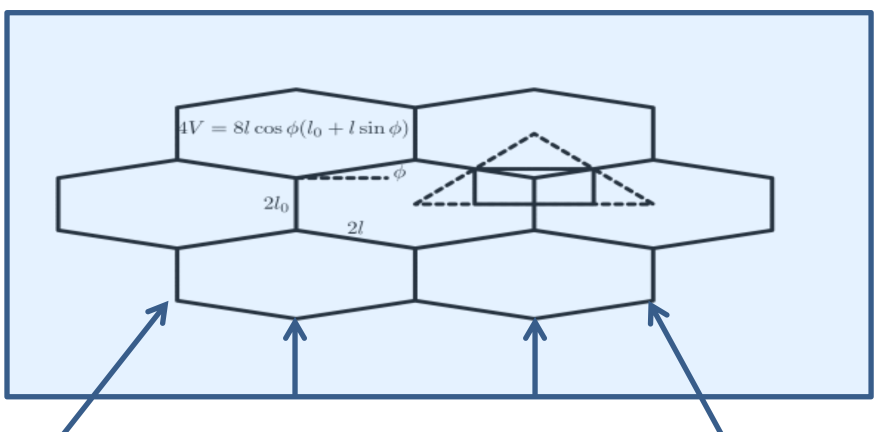
1) **anisotropic inertia** via fine layering

- highly constrained
- large mass issues
- excessive material properties

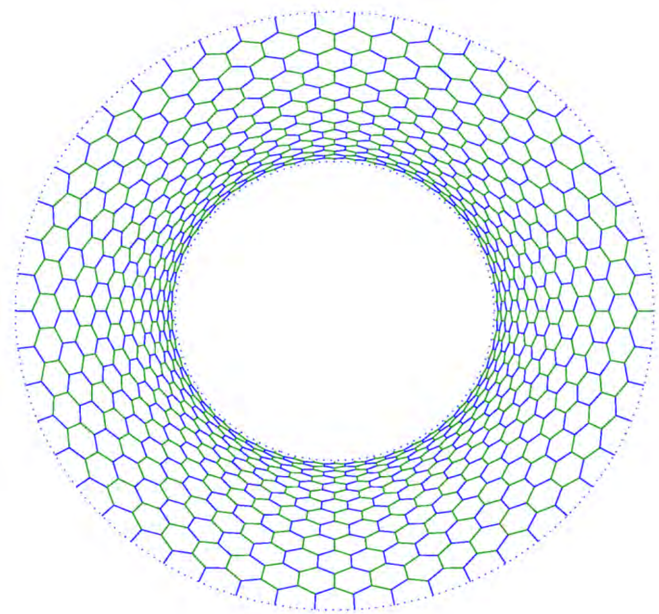


2) **Pentamode Material**

- isotropic inertia
- solid based
- microstructure
- use ideas from composite materials

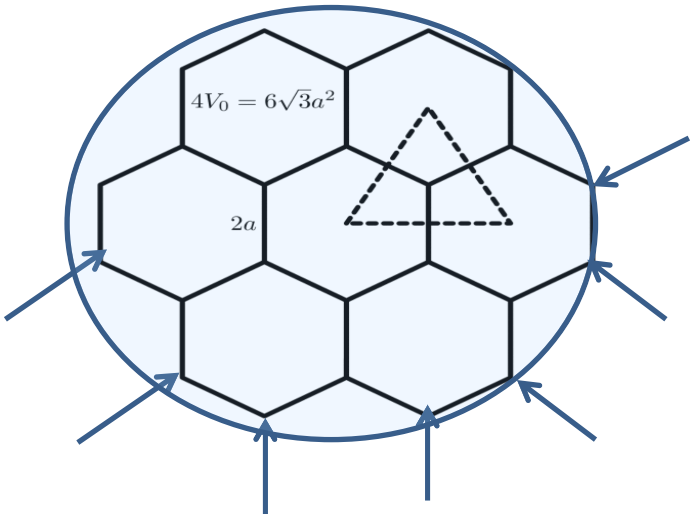


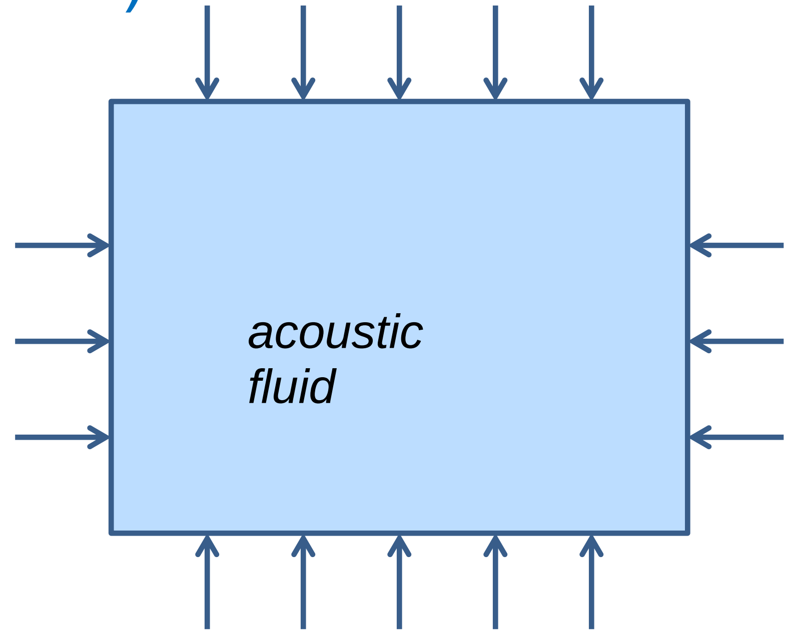
*Idea: unit cell supports only stress **S***

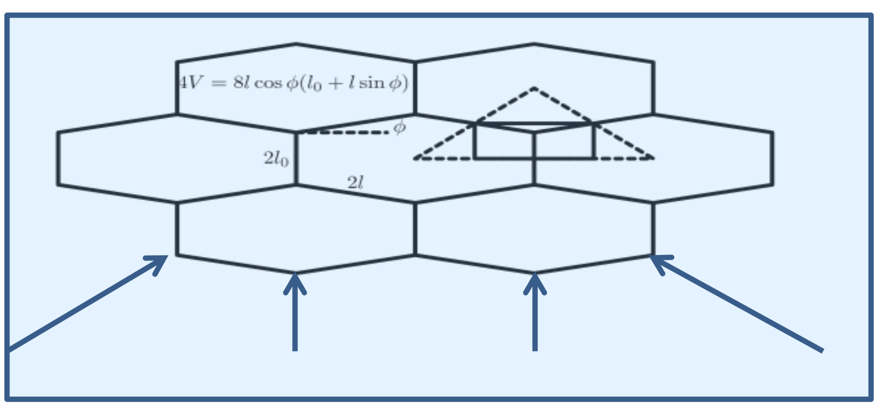


force/truss structure for 2D cloak

Pentamode Material (possible) microstructures







Fluid only supports stress $-p\,\mathbf{I}$

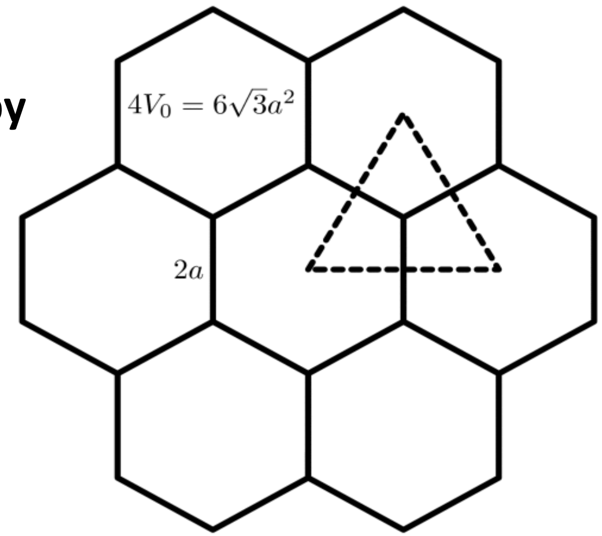
PM supports only stress $\mathbf{S}$

Idea: create unit cell that supports $\mathbf{S}$

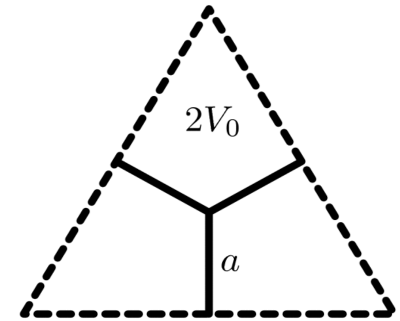
(elements indicate forces)

isotropic and anisotropic networks

isotropy



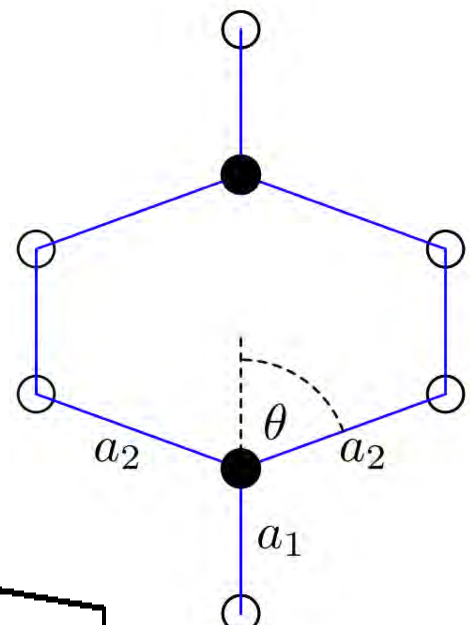
$$\boldsymbol{I} = \sum_{j=0}^{d} \boldsymbol{a}_j \otimes \boldsymbol{a}_j$$



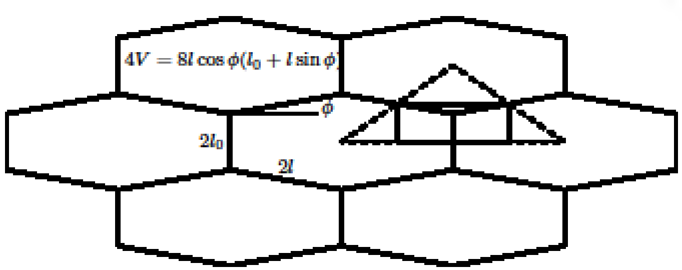
$$\boldsymbol{a}_0 = a(0, -1), \quad \boldsymbol{a}_1 = a(\tfrac{\sqrt{3}}{2}, \tfrac{1}{2})$$
$$\boldsymbol{a}_2 = a(-\tfrac{\sqrt{3}}{2}, \tfrac{1}{2}), \quad a = \sqrt{\tfrac{2}{3}}$$

anisotropy

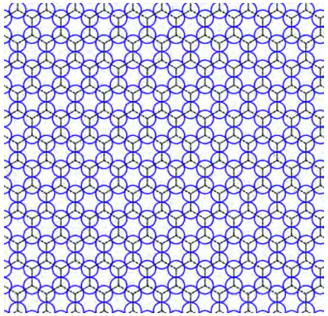
$$\boldsymbol{S} = \boldsymbol{S}^{1/2}\boldsymbol{I}\boldsymbol{S}^{1/2}$$



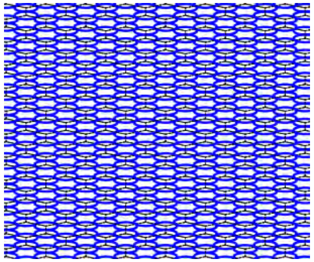
$$\boldsymbol{S} = \sum_{j=0}^{d} \boldsymbol{a}'_j \otimes \boldsymbol{a}'_j$$



$$\boldsymbol{a}'_j = \boldsymbol{S}^{1/2}\boldsymbol{a}_j$$

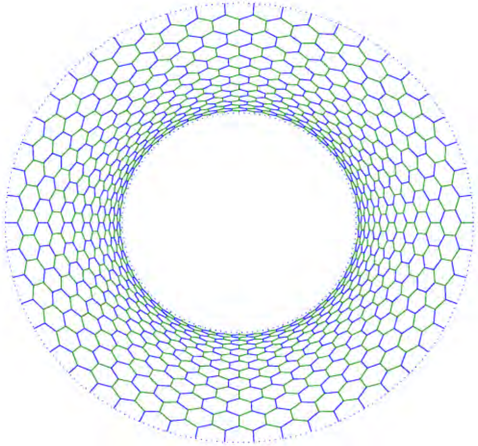


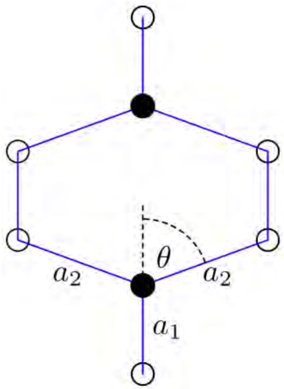
a generic material?



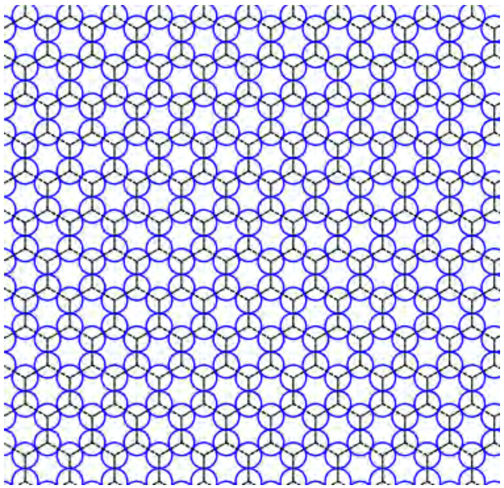
some desired/necessary properties:

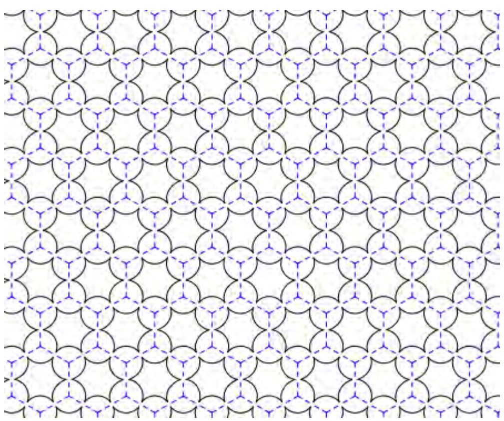
- **transparent** to underwater sound in its "base state"
- **density** of water in its base state   (conservation of mass property)
- effective **bulk modulus** of water in base state
- "**deformable**" to achieve anisotropy
- **maximize** potential cloaked space



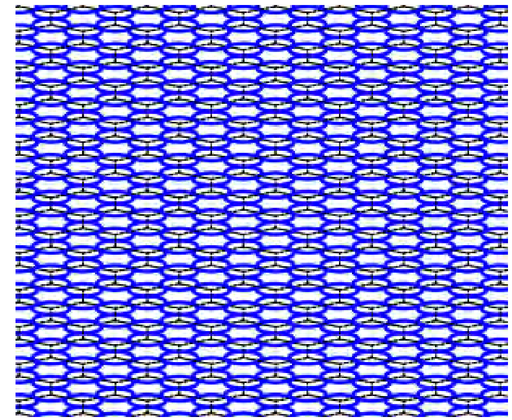


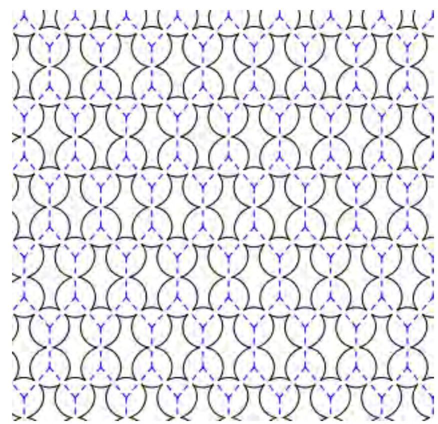
pentamode material (PM) microstructure possibilities

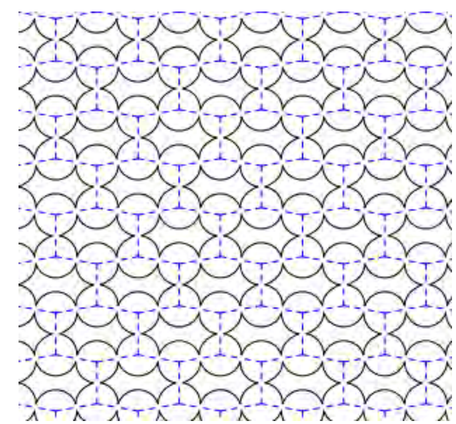




isotropic

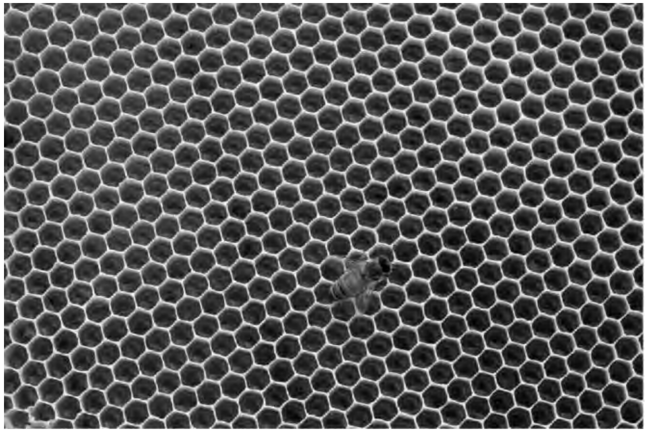


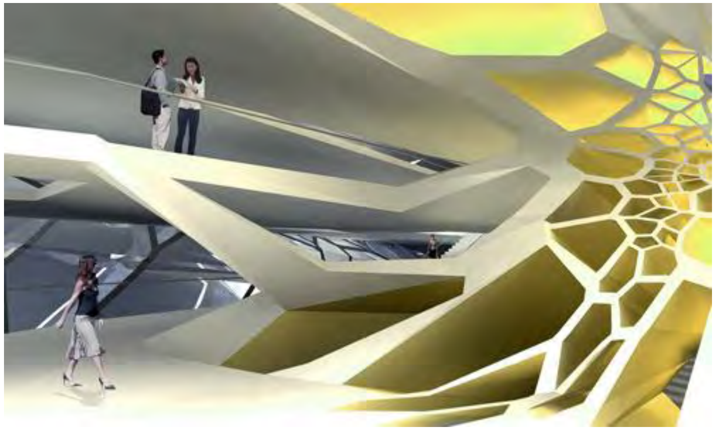


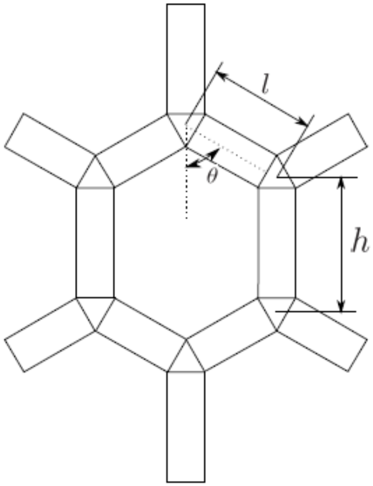


anisotropic

cellular foam structures







extensional and bending compliances
(inverse of stiffness)

$$M_a = \int_0^{a/2} \frac{\mathrm{d}\,x}{EA}, \quad N_a = \int_0^{a/2} \frac{x^2\,\mathrm{d}\,x}{EI}$$

$$\boldsymbol{C} = \begin{pmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{pmatrix} = \begin{pmatrix} \frac{E_1^*}{1-\nu_{12}^*\nu_{21}^*} & \frac{\nu_{21}^* E_1^*}{1-\nu_{12}^*\nu_{21}^*} & 0 \\ \frac{\nu_{12}^* E_2^*}{1-\nu_{12}^*\nu_{21}^*} & \frac{E_2^*}{1-\nu_{12}^*\nu_{21}^*} & 0 \\ 0 & 0 & G_{12}^* \end{pmatrix}$$

effective elastic moduli

$$E_1^* = \frac{l \sin\theta}{2b(h + l\cos\theta)(N_l' \cos^2\theta + M_l \sin^2\theta)},$$

$$E_2^* = \frac{h + l\cos\theta}{2bl \sin\theta(N_l' \sin^2\theta + M_l \cos^2\theta + 2M_h)},$$

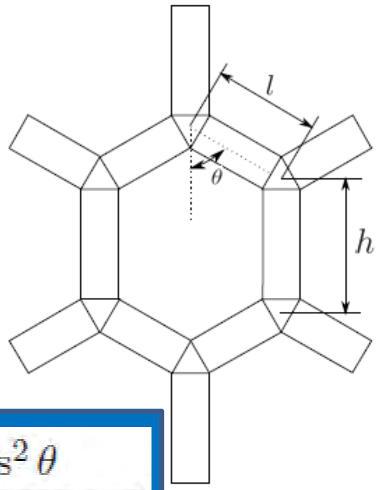
$$\nu_{12}^* = \frac{l \sin\theta(N_l' - M_l)\cos\theta \sin\theta}{(h + l\cos\theta)(N_l' \cos^2\theta + M_l \sin^2\theta)},$$

$$\nu_{21}^* = \frac{(h + l\cos\theta)(N_l' - M_l)\cos\theta \sin\theta}{l \sin\theta(N_l' \sin^2\theta + M_l \cos^2\theta + 2M_h)},$$

$$G_{12}^* = \frac{(h + l\cos\theta) l \sin\theta}{2b((h^2 N_l' + 2l^2 N_h')\sin^2\theta + M_l(l + h\cos\theta)^2)},$$

(Kim & Hassani 2003)

cellular foam as a Pentamode Material



thin members have small bending stiffness, leading to (approximately)

$$\boldsymbol{C} = \begin{pmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{pmatrix} = C_0 \begin{pmatrix} \alpha & 1 & 0 \\ 1 & \frac{1}{\alpha} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad C_0 = \frac{\sin\theta\cos\theta}{2b(M_l + 2M_h \sin^2\theta)}$$

$$\alpha = \frac{l\cos^2\theta}{(h + l\sin\theta)\sin\theta}$$

acoustic fluid: $\boldsymbol{C} = \begin{pmatrix} K & K & 0 \\ K & K & 0 \\ 0 & 0 & 0 \end{pmatrix}$ ⇒ Cellular structure has PM form with unique stress

$$\boldsymbol{\sigma} = -p \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix}$$

bottom line: cellular foam-like structure with **thin** members can provide the PM stiffness

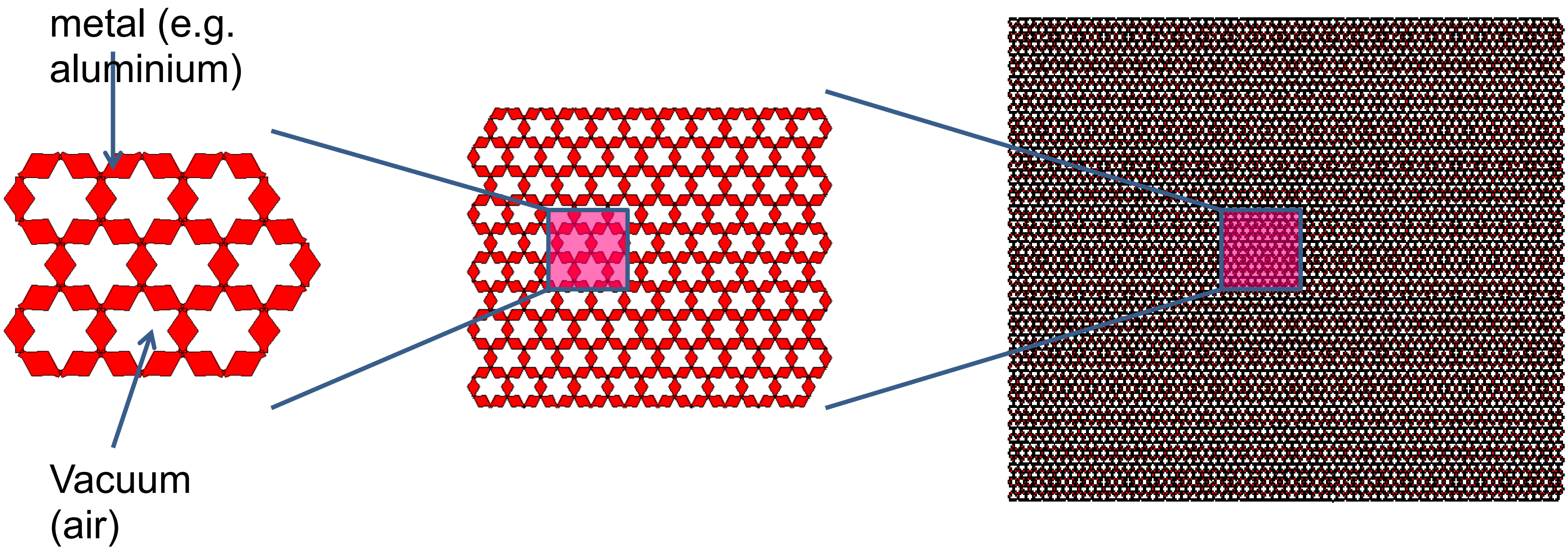
$$C_{66} \neq 0 \text{ but is small, } C_{66}/C_0 = \mathrm{O}\left(\frac{M}{N}\right) = \frac{\text{bending stiffness}}{\text{extensional stiffness}}$$

with relatively small shear rigidity

thin members = more potential cloaking space

……….. stiff, dense

Metal Water



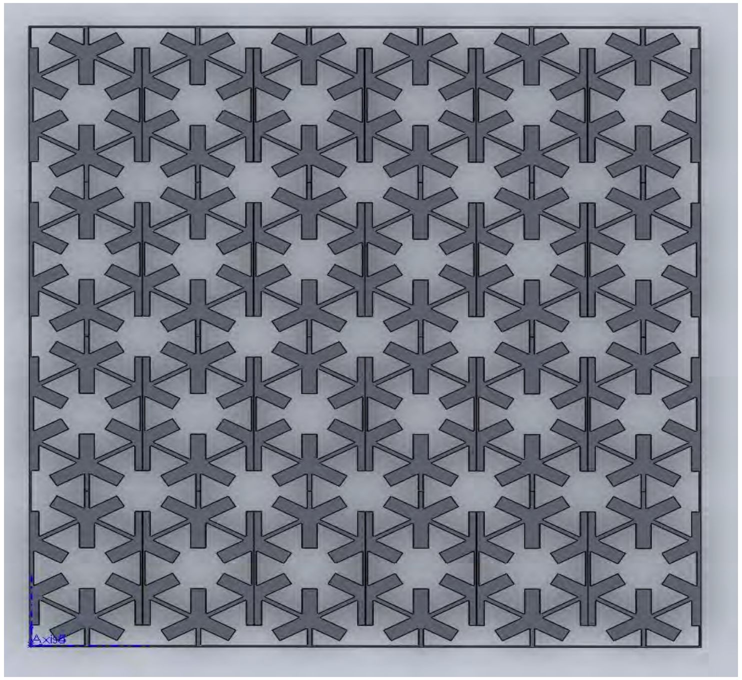
Idea: make water from **metal** foam

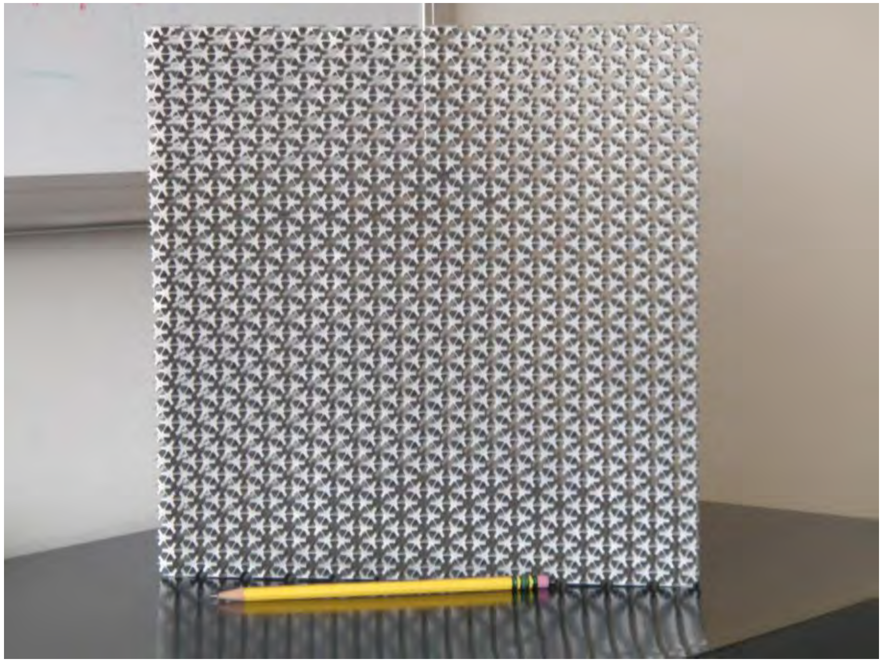
e.g. start with block of Al, remove solid metal to get:

a) effective density of water
b) effective isotropic elasticity with bulk modulus of water, **small shear modulus**

MW prototypes

- **Have manufactured prototypes from Aluminum plate stock using a water-jet process.**
- In-water transmission testing of the full part completed.

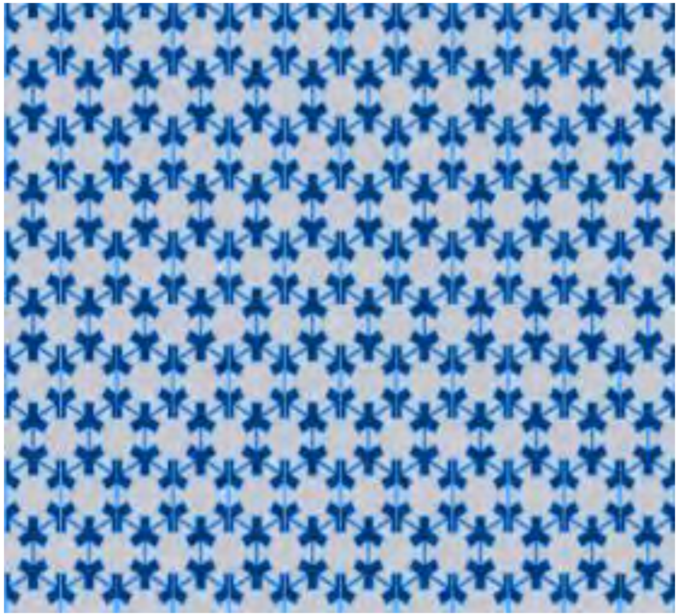


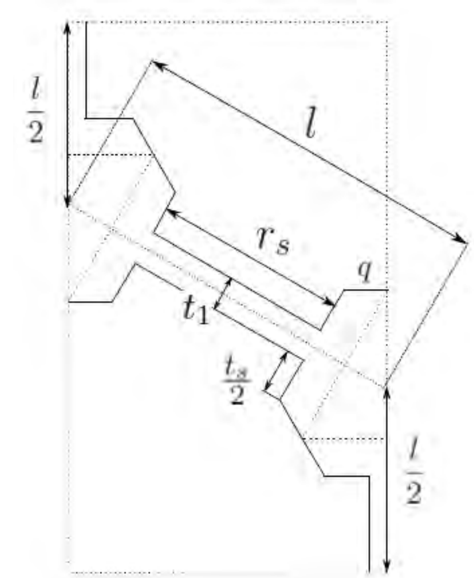


Bulk modulus = 2.25 Gpa

Density = 1000 kg/m^3

Shear modulus = 0.065 GPa



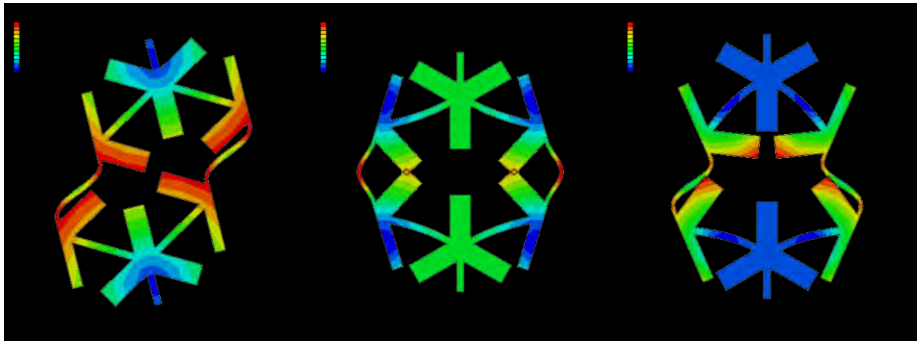


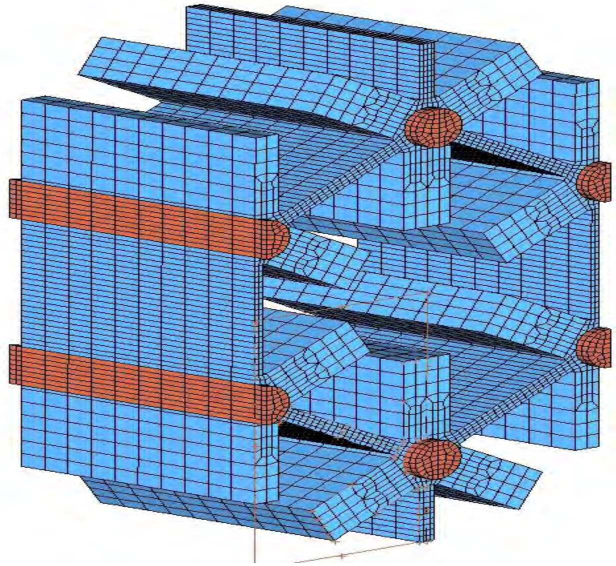
FEM

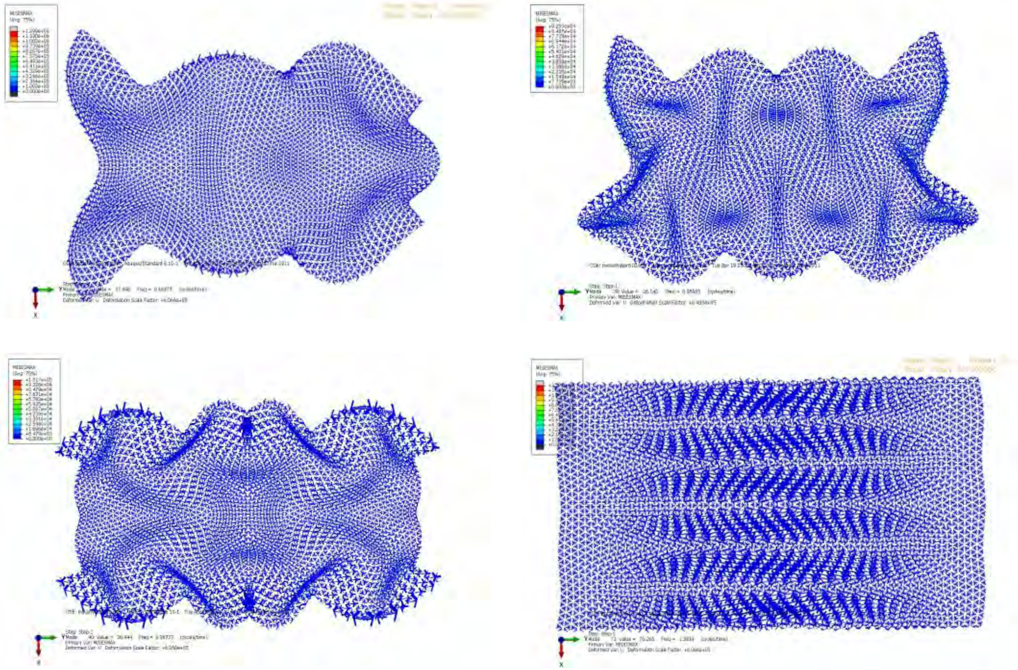
$$M_a = \int_0^{a/2} \frac{\mathrm{d}\,x}{EA}, \quad N_a = \int_0^{a/2} \frac{x^2\,\mathrm{d}\,x}{EI}$$

islands : inertial role only - denser is better
isthmuses : elastic role only - stiffer is better

→ **metal**



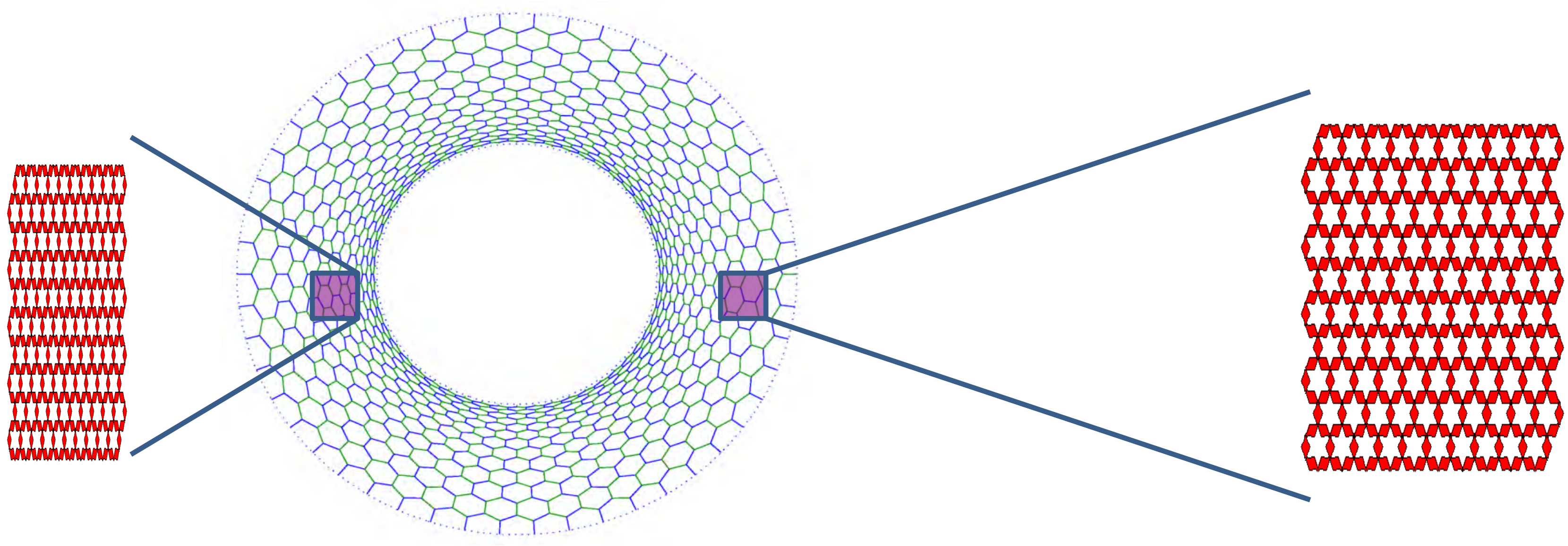




- **Eigenanalysis of the Metal Water design indicates shear-related modes.**
- As expected, since the design cannot be and is no perfectly Pentamode.
- **Shear modes are high-k, low-frequency, easily damped, not expected to affect test results.**
- Transient explicit analysis of the Metal Water design indicates that the shear-related deformation modes do not have a strong effect on wave propagation, when a sample is ensonified using a plane wave.

FLEX simulations
(Weidlinger Associates)

Metal Water → cloak

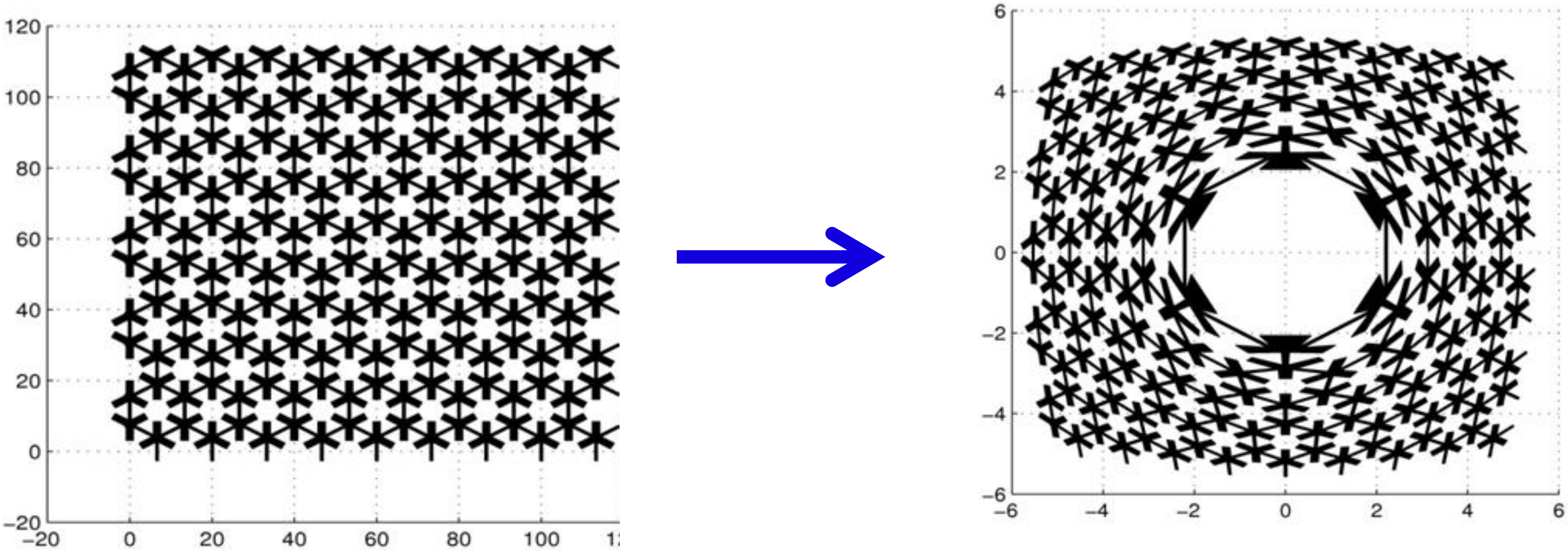


New metafluid = radially compressed version of metallic water

same total mass as uncompressed region

Conservation of cloaked space

conservation of mass = conservation of cloaked space



Heavy metal preferred

PM – Summary

Metal Water offers the potential of solid structures that

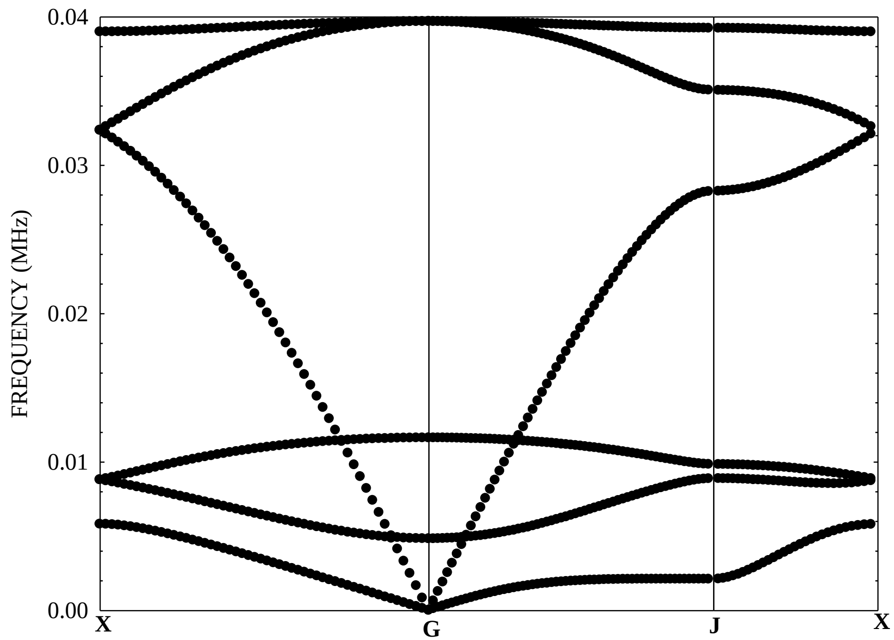
- mimic the acoustic properties of water (isotropic)
- enable transformation acoustics (anisotropic)

using the long wavelength homogenization properties of structured metallic foams as Pentamode materials

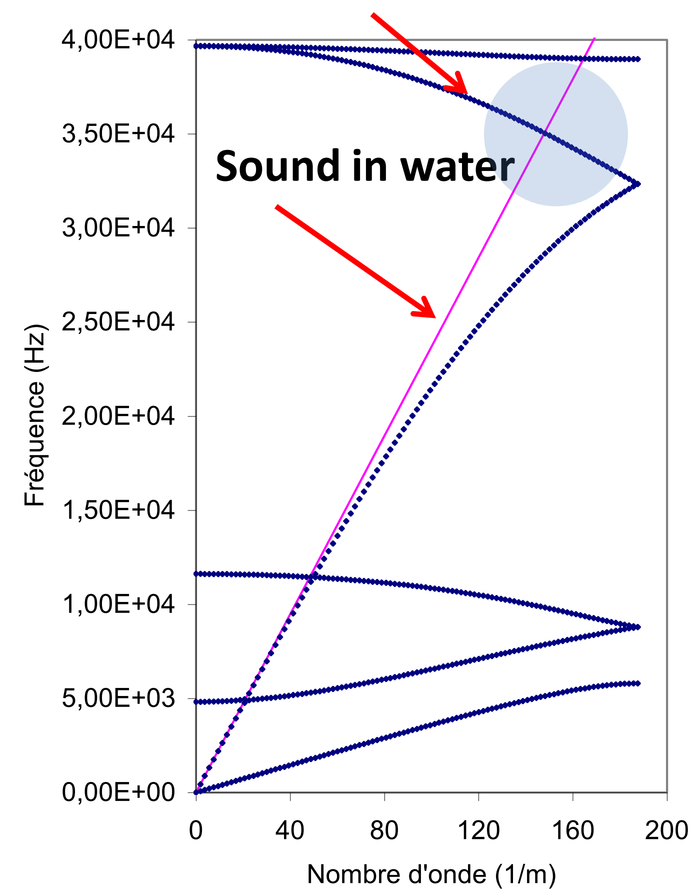
Other uses?

Metal Water: application to negative index materials

Dispersion curve in the first Brillouin zone

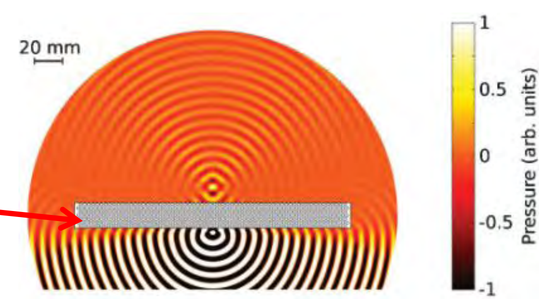


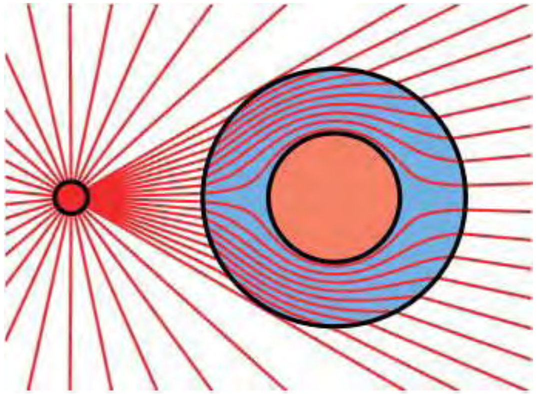
negative group velocity branch



material properties appear
to match with sound in water

Goal: NIM lens with MW
using negative group veocity branches





Introduction: metamaterials

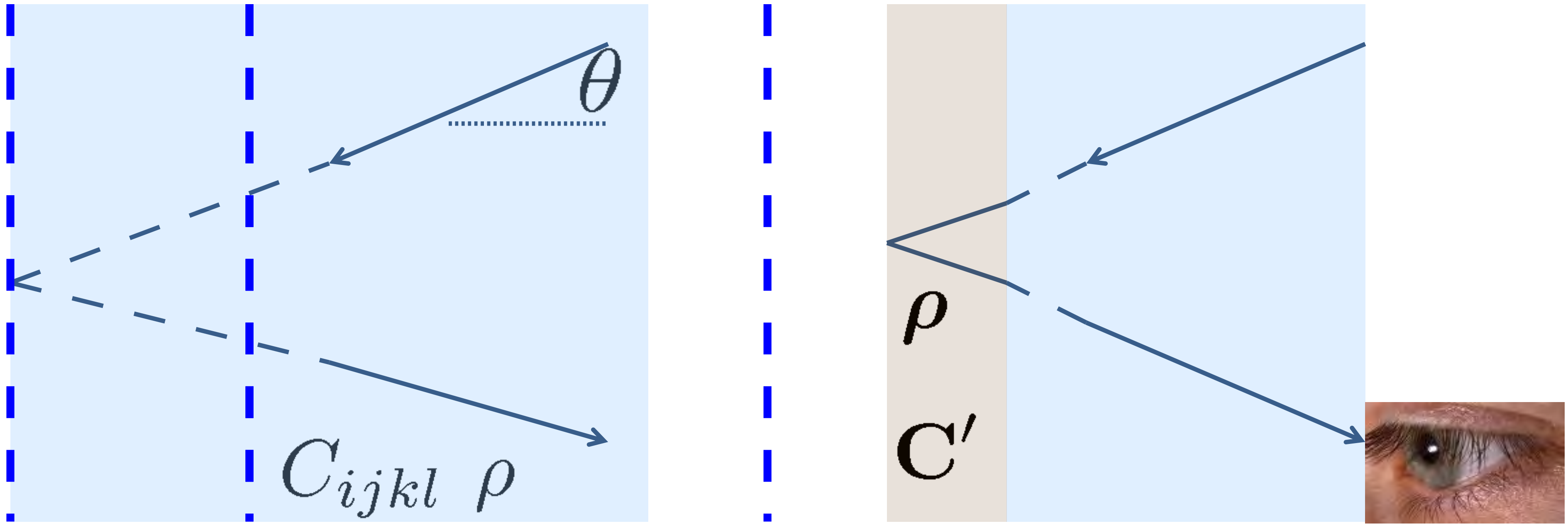
Phononic crystals – engineering the band gap
waves in periodic systems
examples of phononic crystals

Acoustic cloaking – engineering the impossible
transformation acoustics
1D, 2D, cylindrical
inertial materials
Pentamode materials
cloaking elastic waves

Metatheory for metamaterials?

cloaking of elastic waves

cloaking in elasticity ?



miraging and **cloaking**, work in principle - but require materials that are not "elastic"

- Density can always be made isotropic
- Stress is usually not symmetric, ***Cosserat*** materials are necessary
- normal elastic materials can provide approximate cloaking

elastic transformation theory

- transformation $\mathbf{X} \to \mathbf{x} \qquad F_{iJ} = \frac{\partial x_i}{\partial X_J}$

- as in **acoustics**, the materials are not unique. They can be characterized by how the displacement transforms $\mathbf{U} \to \mathbf{u} = \mathbf{A}\mathbf{U}$

- 2 parameters: $\mathbf{F}, \quad \mathbf{A}$

- material generally of **Willis form**, with properties including matrix density that are functions of frequency

- BUT constant isotropic density if $\mathbf{A} = \mathbf{I}$ then moduli of **Cosserat form**

$$\rho^{\text{eff}} = J^{-1}\rho_0, \qquad C^{\text{eff}}_{ijkl} = J^{-1} F_{iI} F_{kK} C^{(0)}_{IjKl}$$

(ANN, ALS 2011)

elastic transformation ⟶ unique nonlinear materials

$$\rho^{\mathrm{eff}} = J^{-1}\,\rho_0, \qquad C^{\mathrm{eff}}_{ijkl} = J^{-1}\,F_{iI}F_{kK}\,C^{(0)}_{IjKl}$$

(ANN, ALS 2011)

transformed elastic moduli are tangent moduli for a hyperelastic material with strain energy

$$W = \frac{1}{2}(U_{j\alpha} - \delta_{j\alpha})(U_{l\beta} - \delta_{l\beta})C^{(0)}_{\alpha j\beta l}$$

under a state of (pre)stress

$$\sigma^{\mathrm{pre}}_{ij} = J^{-1}F_{i\alpha}(F_{l\beta} - \delta_{l\beta})C^{(0)}_{j\alpha\beta l}$$

- equilibrium of the pre-stress constrains the transformation **F** to satisfy

$$C^{(0)}_{j\alpha\beta l}x_{l,\alpha\beta} = 0$$

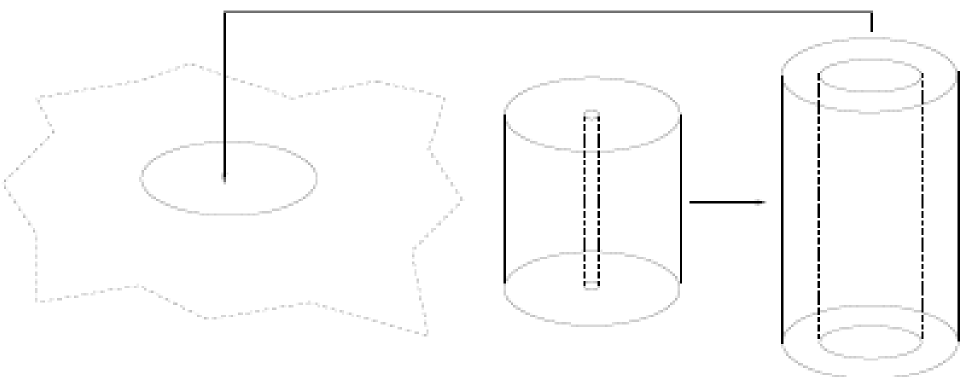
- for **acoustics**, this includes all radially symmetric (d=2,3) transformations

cloaking in elasticity

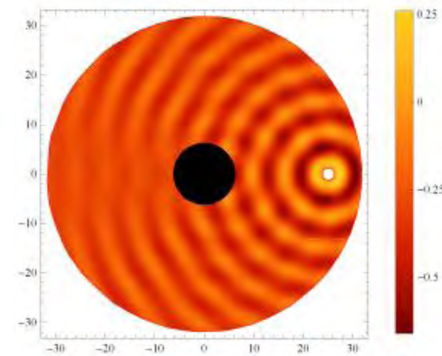
this means elastic waves can be cloaked using a material with a **unique finite strain energy function**

expand small holes to finite size: the deformed solid has small-on-large moduli *exactly* those required from the transformation

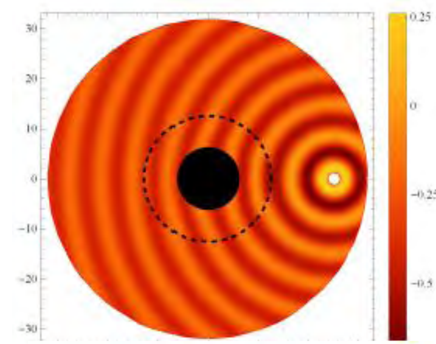
ex. SH waves



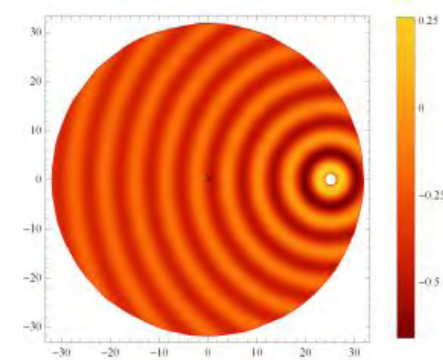
bulk material initial hole expanded



line source
- rigid cylinder
- no cloak



pressurize small hole to size of cylinder, and insert rigid body

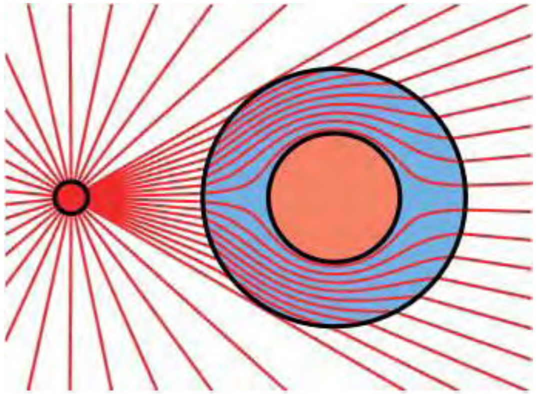


scattering from original hole

neo-Hookean strain energy function

$$W = \frac{\mu}{2}(\lambda_r^2 + \lambda_\theta^2 + \lambda_z^2 - 3)$$

μ = original shear modulus



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cloaking elastic waves

Metatheory for metamaterials?

A metatheory for metamaterials?

Phononic crystals:

homogenization of periodic systems at finite frequency & wavelength
leads to effective material properties consistent with the

Willis constitutive model:

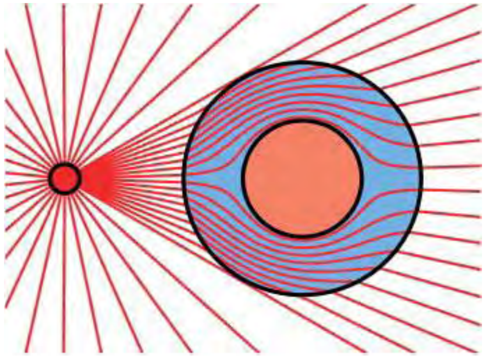
$$\dot{\mathrm{p}}^{\mathrm{eff}} = \mathrm{div}\,\boldsymbol{\sigma}^{\mathrm{eff}} \qquad \begin{pmatrix} \boldsymbol{\sigma}^{\mathrm{eff}} \\ \mathrm{p}^{\mathrm{eff}} \end{pmatrix} = \begin{pmatrix} \mathrm{C}^{\mathrm{eff}} & \mathrm{S}^{\mathrm{eff}} \\ -\mathrm{S}^{\mathrm{eff}+} & \rho^{\mathrm{eff}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\epsilon}^{\mathrm{eff}} \\ \dot{\mathrm{u}}^{\mathrm{eff}} \end{pmatrix}$$

(Willis 2010, Milton and Willis 2006, Shuvalov et al. 2010, Srivistava & Nemat-Nasser 2011)

$$e^{ik_1 a_1 \mathbf{A}_1} e^{ik_2 a_2 \mathbf{A}_2} \quad ? =? \quad e^{ik_{\mathrm{eff}} a \mathbf{A}_{\mathrm{eff}}}$$

Acoustic and elastic cloaking:

The most general constitutive model that is invariant under “transformation” is the
Willis constitutive model. (Milton, Briane, Willis, 2007, ANN 2011)



in closing

metamaterials, especially cloaking devices, combine a rich mixture of topics

- wave mechanics/physics
- continuum mechanics
- differential geometry
- anisotropic elasticity
- finite elasticity
- materials science
- fabrication issues
- computational methods
- etc.

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