Turbulence in Pipes: The Moody Diagram and Moore’s Law

Alexander Smits
Princeton University
ASME Fluids Engineering Conference
San Diego, July 30-August 2, 2007
Thank you

• Mark Zagarola, Beverley McKeon, Rongrong Zhao, Michael Shockling, Richard Pepe, Leif Langelandsvik, Marcus Hultmark, Juan Jimenez
• James Allen, Gary Kunkel, Sean Bailey
• Tony Perry, Peter Joubert, Peter Bradshaw, Steve Orszag, Jonathan Morrison, Mike Schultz
Lewis Ferry Moody 1880-1953

- Professor of Hydraulic Engineering at Princeton University, 1930-1948

- “Friction factors for pipe flow,” Trans. of the ASME, 66, 671-684, 1944

(from Glenn Brown, OSU)
The Moody Diagram

\[ \lambda = \frac{-\frac{dp}{dx}D}{\frac{1}{2} \rho \bar{U}^2} \]

Laminar

Increasing roughness k/D

Smooth pipe (Blasius)

Smooth pipe (Prandtl)

\[ Re = \bar{U}D/\nu \]
Outline

- Smooth pipe experiments
  - \( \text{Re}_D = 31 \times 10^3 \) to \( 35 \times 10^6 \)
- Rough pipe experiments
  - Smooth to fully rough in one pipe
  - Honed surface roughness
  - Commercial steel pipe roughness
- A new Moody diagram(s)?
- Predicting arbitrary roughness behavior
  - Theory
  - Petascale computing
High Reynolds number facilities

National Transonic Facility, NASA-LaRC

80’ x 120’
NASA-Ames
High Reynolds Number in the lab:
Compressed air up to 200 atm as the working fluid

Princeton/DARPA/ONR Superpipe:
Fully-developed pipe flow
\( \text{Re}_D = 31 \times 10^3 \text{ to } 35 \times 10^6 \)
Standard velocity profile

\[ U^+ = U/u_\tau \]

Inner variables

Overlap region

\[ U^+ = \frac{1}{\kappa} \ln y^+ + B \]

\[ y^+ = yu_\tau / v \]

\[ \lambda = \frac{4\tau_w}{\frac{1}{2} \rho U^2} = 8 \left( \frac{u_\tau}{U} \right)^2 \]

\[ u_\tau = \sqrt{\tau_w / \rho}, \]
Similarity analysis for pipe flow

Incomplete similarity (in Re) for inner & outer region

\[ \frac{U}{u_\tau} = U^+ = f\left(\frac{yu_\tau}{v}, \frac{Ru_\tau}{v}\right) = f(y^+, R^+) \quad \text{Inner scaling} \]

\[ \frac{U_{CL} - U}{u_o} = g\left(\frac{y}{R}, \frac{Ru_\tau}{v}\right) = g(\eta, R^+) \quad \text{Outer scaling} \]

Complete similarity (in Re) for inner & outer region

\[ U^+ = f\left(\frac{yu_\tau}{v}\right) = f(y^+) \quad \text{Inner} \]

\[ \frac{U_{CL} - U}{u_o} = g\left(\frac{y}{R}\right) = g(\eta) \quad \text{Outer} \]
Overlap analysis: two velocity scales

At low Re:

\[ \frac{u_o}{u_\tau} = h(R^+) \]

> Match velocities and velocity gradients ⇒ power law

At high Re:

\[ \frac{u_o}{u_\tau} = \text{constant} \]

> Match velocities and velocity gradients ⇒ power law
> Match velocity gradients ⇒ log law

\[ U^+ = \frac{1}{\kappa} \ln y^+ + B \]
Pipe flow inner scaling

\[ U^+ \approx 8.70(y^+)^{0.137} \]

\[ U^+ = \frac{1}{\kappa} \ln y^+ + B \]

\[ U^+ = y^+ \]
Smooth pipe summary

- Log law only appears at sufficiently high Reynolds number
- New log law constants: $\kappa=0.421$, $B=5.60$ (cf. 0.41 and 5.0)
- Spalart: $\Delta\kappa = 0.01$, gives $\Delta C_D=1\%$ at flight Reynolds numbers
- New outer layer scaling velocity for “low” Reynolds number

What about the friction factor? Need to integrate velocity profile.

$$\lambda = \frac{4\tau_w}{\frac{1}{2}\rho U^2} = 8 \left(\frac{u_\tau}{U}\right)^2$$

$$U^+ = \frac{1}{\kappa} \ln y^+ + B$$
Gottingen, Germany

Theodor von Kármán

Ludwig Prandtl

Johann Nikuradse, 1933
(from Glenn Brown)
Prandtl’s “Universal Friction Law”

With $\kappa = 0.4$, and $B = 5.5$ (Nikuradse), and $C_3 = C_4 = 0$:

$$\frac{1}{\sqrt{\lambda}} = 2.035 \ln Re \sqrt{\lambda} - 0.913$$

Prandtl:

$$\frac{1}{\sqrt{\lambda}} = 2.0 \log Re_D \sqrt{\lambda} - 0.8$$
Nikuradse’s data

\[ \frac{1}{\sqrt{\lambda}} = 2.0 \log Re_D \sqrt{\lambda} - 0.8 \]

Figure 1.2: Comparison of the Blasius relationship (---), Prandtl’s friction factor relationship (\(-\cdot-\)) and the friction factor data of Nikuradse (which was used to formulate the empirical expression).
Superpipe results

Prandtl (1935)

\[
\frac{1}{\sqrt{\lambda}} = 2.0 \log Re_D \sqrt{\lambda} - 0.8
\]

McKeon et. al. (2004)

\[
\frac{1}{\sqrt{\lambda}} = 1.930 \log(Re \sqrt{\lambda}) - 0.537
\]
Two complementary experiments

Princeton Superpipe
Oregon

λ

Re

10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8
Roughness

- How do we know the “smooth” pipe was really smooth at all Reynolds numbers?
- Were the higher friction factors at high Reynolds numbers evidence of roughness?

- What is $k$?
  - rms roughness height: $k_{rms}$
  - equivalent sandgrain roughness: $k_s$

- Nikuradse’s rough pipe experiments (sandgrain roughness)
  - $k_s^+ < 5$, smooth
  - $5 < k_s^+ < 70$, transitionally rough
  - $k_s^+ > 70$, fully rough

\[
k_s^+ = \frac{k_s u_T}{\nu}
\]
Nikuradse's sandgrain experiments

- Transition from smooth to fully rough included inflection

- "Quadratic Resistance" in fully rough regime - Reynolds number independence
Colebrook and the Moody Diagram

• Data from Colebrook & White (1938), Colebrook (1939)
• Tested various roughness types
  – Large and small elements
  – Sparse and dense distributions
• Studied many different pipes with “commercial” roughness
• Nikuradse sand-grain trend with inflection deemed irregular
• Focus on the behavior in the transitional roughness regime

Cyril F. Colebrook
(from Glenn Brown)
Lewis Moody
\[ \lambda = \frac{- \frac{dp}{dx} D}{\frac{1}{2} \rho U^2} \]

\[ \sqrt{\lambda} = -2 \log \left( \frac{k_s}{3.71D} + \frac{2.51}{\text{Re} \sqrt{\lambda}} \right) \]

Colebrook transitional roughness function

\[ \text{Re} = \bar{U} D / \nu \]
Where did Colebrook’s function come from?

- Colebrook (1939)
Colebrook & White boundary layer results

\[ \lambda \] friction factor

\[ \text{Re}_D \]
New experiments on roughness

• Use Superpipe apparatus to study different roughness types by installing different rough pipes
• Advantage: able to cover regime from smooth to fully rough with one pipe
• Honed surface roughness
  – To help establish where Superpipe data becomes rough (10 x 10^6, or 28 x 10^6, or 35 x 10^6, or what?)
  – To help characterize an important roughness type (honied and polished finish) (k_s = 6k_{rms}, k_s = 3k_{rms}?)
• Commercial steel pipe roughness
  – Most important surface for industrial applications
Honed surface finish

"Smooth" pipe, 6μin

\[
\frac{k_{RMS}}{D} = 1.16 \times 10^{-6} \quad k_{RMS} = 0.15\mu m
\]

Honed rough pipe, 98μin

\[
\frac{k_{RMS}}{D} = 1.94 \times 10^{-5} \quad k_{RMS} = 2.5\mu m
\]
Results in smooth regime
Results: transitional/rough regime
Inner scaling - all profiles

\[ \Delta U^+ \text{ Hama roughness function} \]
Therefore “smooth” Superpipe was smooth for $\text{Re}_D \leq 21 \times 10^6$
Friction factor results for rough pipe

\[ \frac{k_{RMS}}{D} = 1.94 \times 10^{-5} \]
\[ \frac{k_s}{D} = 5.81 \times 10^{-5} \]
\[ \lambda_{rough} = 0.0108 \]
Revised resistance diagram for honed surfaces

[Diagram showing various curves representing different roughness levels, labeled as "Smooth" and "Rough" with respective legend markers.]
Commercial steel surface roughness

Honed rough pipe, 98μin

\[ \frac{k_{RMS}}{D} = 1.94 \times 10^{-5} \quad k_{RMS} = 2.5 \mu m \]

Commercial steel rough pipe, 195μin

\[ \frac{k_{RMS}}{D} = 3.82 \times 10^{-5} \quad k_{RMS} = 5.0 \mu m \]
Sample 2: non-rust spot
Sample 2: rust spot
Velocity profiles: inner scaling

Velocity profile, inner scaling, $Re < 220 \times 10^3$
Velocity profiles: inner scaling

Velocity profiles, inner scaling, $300 \times 10^3 < \text{Re} < 700 \times 10^3$
Velocity profiles: inner scaling

Velocity profile, inner scaling, $Re > 800 \times 10^3$
Hama roughness function

\[ \Delta U^+ \quad k_S^+ \]

Colebrook transitional roughness

commercial steel pipe

honed surface roughness
Commercial steel pipe friction factor

$k_s = 1.5k_{rms}$
Moody diagram for commercial steel pipe
Rough pipe summary

- Honed surface roughness
- Smooth $\rightarrow$ transitional $\rightarrow$ fully rough
- $k_{rms}/D = 19 \times 10^{-6}$
- Pipe L/D = 200
- $Re_D = 57 \times 10^3$ to $21 \times 10^6$
- Smooth for $k_s^+ < 3.5$
- $k_s = 3.0k_{rms}$
- Inflectional friction factor not monotonic (Nikuradse not Colebrook)

- Commercial steel pipe roughness
- Smooth $\rightarrow$ transitional $\rightarrow$ fully rough
- $k_{rms}/D = 38 \times 10^{-6}$
- Pipe L/D = 200
- $Re_D = 93 \times 10^3$ to $20 \times 10^6$
- Smooth for $k_s^+ < 3.1$
- $k_s = 1.5k_{rms}$ (instead of $3.5k_{rms}$!)
- Friction factor monotonic (but not Colebrook)
Why the Moody Diagram needs updating

- Prandtl’s universal friction factor relation is not universal (breaks down at higher Reynolds numbers: >3 x 10^6)
- Transitional roughness regime is represented by Colebrook’s transitional roughness function using an equivalent sandgrain roughness, which takes no account of individual roughness types
- Honed surfaces are inflectional not monotonic
- Commercial steel pipe monotonic but not Colebrook

- The limitations of the Moody Diagram were well-known (e.g., Hama), but no match for text book orthodoxy
Where do we go from here?

• More experiments, more data analysis?
  – Schultz and Flack

• A predictive theory?
  – Gioia and Chakraborty

• Petascale computing?
  – Moser, Jimenez, Yeung
Goia and Chakraborty’s (2006) model

- Model the energy spectrum in the inertial and dissipative ranges
- Use the energy spectrum to estimate the speed of eddies of size $s$
- Model the shear stress on roughness element of size $s$ as $\tau \approx \rho \bar{U} u_s$
- Hence $\lambda \propto u_s / \bar{U}$, then integrate across all scales to find $\lambda$
Prospects for Computation: Moore’s Law

Intel co-founder Gordon Moore

April, 1965

2005
Petascale computing

• Earth Simulator (2004): $36 \times 10^{12}$ flops peak
  – DNS of $4096^3$ isotropic turbulence

• Petascale computing (2007): Blue Gene/P $3 \times 10^{15}$ flops peak
  – Remarkable resource, but what questions can it answer?

• Example: DNS of channel flow
  – Bob Moser, UT Austin
  – $Re_\tau = u_\tau R / \nu$ (approx $= Re_D / 40$)
Channel flow simulations

- Twin goals: High Reynolds number, Large spatial domain

<table>
<thead>
<tr>
<th>Case</th>
<th>( R_{e\tau} )</th>
<th>( L_x/h )</th>
<th>( L_z/h )</th>
<th>( TU_b/L_x )</th>
<th>( N_x )</th>
<th>( N_z )</th>
<th>( N_y )</th>
<th>( N_t/10^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L550</td>
<td>547</td>
<td>8\pi</td>
<td>4\pi</td>
<td>10</td>
<td>1536</td>
<td>1536</td>
<td>257</td>
<td>1.1</td>
</tr>
<tr>
<td>L950</td>
<td>934</td>
<td>8\pi</td>
<td>3\pi</td>
<td>10</td>
<td>3072</td>
<td>2304</td>
<td>385</td>
<td>2.1</td>
</tr>
<tr>
<td>S1900</td>
<td>1880</td>
<td>\pi</td>
<td>\pi/2</td>
<td>40</td>
<td>768</td>
<td>768</td>
<td>769</td>
<td>0.5</td>
</tr>
<tr>
<td>L2000</td>
<td>2003</td>
<td>8\pi</td>
<td>3\pi</td>
<td>10</td>
<td>6144</td>
<td>4608</td>
<td>633</td>
<td>4</td>
</tr>
</tbody>
</table>


Domain L2000

\[ Re_\tau = \frac{u_\tau h}{\nu} = 2000 \]

\[ x = 8\pi h \approx 50000 + \]

\[ y = 2h \approx 4000 + \]

\[ 3\pi h \approx 19000 + \]
Resources for L2000 (Re_D approx 80,000)

- Processors: 2048 (Power 4)
- Memory: 875 GB (~ 430 MB/processor)
- Restart File Size ~ 64GB (2 DOF/mode)
- Total simulation time: ~ 122 days, or 6 \times 10^6 \text{ proc. hrs}

How much higher can we go?
How much higher need we go?
To this level of approximation:

\[
\frac{\partial u^+}{\partial y^+} \bigg|_i \approx \frac{1}{\kappa y^+} + \epsilon \alpha + \frac{\epsilon \beta}{y^+} \quad \text{for } y^+ \gg 1
\]

\[
\frac{\partial u^+}{\partial \tilde{y}} \bigg|_o \approx \frac{1}{\kappa \tilde{y}} + \alpha + \frac{\epsilon \beta}{\tilde{y}} \quad \text{for } \tilde{y} \ll 1
\]

Integrate to obtain:

\[
u^+_i = \left( \frac{1}{\kappa} + \frac{\beta}{Re_{\tau}} \right) \ln y^+ + \frac{\alpha y^+}{Re_{\tau}} + B_i
\]

\[
u^+_o - u^+_c = \left( \frac{1}{\kappa} + \frac{\beta}{Re_{\tau}} \right) \ln \tilde{y} + \alpha \tilde{y} + B_o
\]
Comparison with DNS channel data

\[ u_i^+ = \left( \frac{1}{\kappa} + \frac{\beta}{Re_\tau} \right) \ln y^+ + \frac{\alpha y^+}{Re_\tau} + B_i \]

\[ u_o^+ - u_c^+ = \left( \frac{1}{\kappa} + \frac{\beta}{Re_\tau} \right) \ln \tilde{y} + \alpha \tilde{y} + B_o \]

\[ \alpha \approx 1.0 \quad \kappa \approx 0.4 \quad \beta \approx 150 \]

- Reasonable approximation for \( y^+ > 300 \) and \( \tilde{y} < 0.45 \)

- First convincing overlap at \( Re_\tau = 2000 \), need \( Re_\tau > 4000 \)
From $\text{Re}_\tau = 2000$ to 5000

- Just high enough to start seeing the asymptotic approach to $Re \to \infty$
  - “Log Layer”
  - Inner and outer scale separation
  - 2-D version of $1/k_x$ spectrum

- $Re_\tau \approx 5000$ will be high enough to confirm asymptote and be convincing

- $L_x \approx 40\delta$ will be a large enough domain to capture full range of scales
Resource requirements

- **Computer Time (Peta Flop Hours – PFH)**

\[ PFH = C_t \tilde{L}_t \tilde{L}_x^2 \tilde{L}_z Re_{\tau}^4 \quad C_t = 1.6 \times 10^{-16} \]

- **Storage (Terabytes – TB)**

\[ TB = C_s \tilde{L}_x \tilde{L}_z Re_{\tau}^3 \quad C_s = 4.3 \times 10^{-13} \]

<table>
<thead>
<tr>
<th>(Re_{\tau})</th>
<th>(\tilde{L}_x)</th>
<th>(\tilde{L}_z)</th>
<th>(\tilde{L}_t)</th>
<th>PFH</th>
<th>ES Hrs</th>
<th>TB</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>16</td>
<td>6</td>
<td>10</td>
<td>1500</td>
<td>96,000</td>
<td>2.64</td>
</tr>
<tr>
<td>5000</td>
<td>8</td>
<td>3</td>
<td>10</td>
<td>193</td>
<td>12,000</td>
<td>0.66</td>
</tr>
<tr>
<td>2000</td>
<td>8</td>
<td>3</td>
<td>10</td>
<td>5</td>
<td>300</td>
<td>0.08</td>
</tr>
<tr>
<td>2000</td>
<td>4</td>
<td>2</td>
<td>10</td>
<td>0.8</td>
<td>50</td>
<td>0.03</td>
</tr>
<tr>
<td>1000</td>
<td>8</td>
<td>3</td>
<td>10</td>
<td>0.3</td>
<td>19</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\(\tilde{L}_x = \frac{L_x}{\pi}\) \quad \(\tilde{L}_z = \frac{L_z}{\pi}\) \quad \(\tilde{L}_t = \frac{L_t U_b}{L_x}\)

- With a sustained Peta-Flop, log-Layer is within reach
Are we done with channels at $\text{Re}_\tau = 5000$?

- Will give about an octave of log-law
- Will display “true” inner and outer regions
- Inadequate for high Reynolds number scaling (need $\text{Re}_\tau > 50,000$)
- What about roughness?
- With a 10 Petaflop machine
  - $\text{Re}_\tau = 5000$ is cheap enough to do experiments
  - Roughness studies?
- Maybe we can do roughness with a teraflop machine (if we are clever)
Conclusions

- Moody diagram should be revised, or used with caution
- Colebrook is pessimistic (makes us look good)
- Transitional roughness behavior not universal: depends on roughness
- Gioia model combined with better surface characterization may lead to predictive theory
- Petascale computing will provide powerful resource for fluids engineering, but maybe we’ll “solve” roughness without it
- A “Golden Age” in the study of wall-bounded turbulence?
Questions??

Osborne Reynolds